Simulating Subatomic Physics on a Quantum Frequency Processor

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How Did This Work Come About?


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TERMINATED CASCADE IS NOT TERMINAL —

Careful phasing of a photonic qubit brings light under control

Controlled color cascade yields better parallelism for photonic qubits.

CHRIS LEE - 2/8/2018, 12:00 PM

PRL **120**, 030502 (2018)

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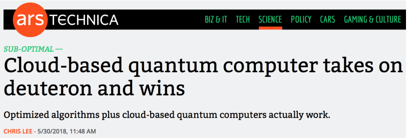
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SUB-OPTIMAL —

Cloud-based quantum computer takes on deuteron and wins

Optimized algorithms plus cloud-based quantum computers actually work.

CHRIS LEE - 5/30/2018, 11:48 AM

PRL **120**, 210501 (2018)

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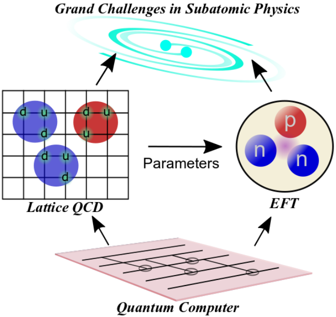
Simulations of Subatomic Many-Body Physics on a Quantum Frequency Processor

Hsuan-Hao Lu[1], Joseph M. Lukens[2], Natalie Klco[3], Martin J. Savage[3], Titus D. Morris[2], Aaina Bansal[4], Andreas Ekström[5], Gaute Hagen[6,4], Thomas Papenbrock[4,6], Andrew M. Weiner[1], and Pavel Lougovski[2,*]

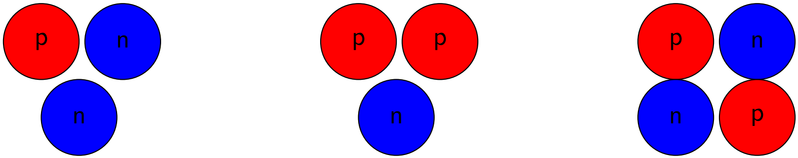
arXiv:1810.03959

Quantum Computing for Subatomic Physics Programme

- *Ab initio* calculations of nucleon and nuclear matter properties from lattice quantum chromodynamics (QCD)
- Effective field theory (EFT) description of nulcei
- Explain astrophysical events at extreme nucleon densities



Nuclear EFT Systems of Interest



^{3}H ^{3}He ^{4}He

Q: Compute the binding energy from an EFT Hamiltonian

$$V \rightarrow \int d\mathbf{r}\Psi^\dagger(\mathbf{r})V_1\Psi(\mathbf{r}) + \int d\mathbf{r}d\mathbf{r}'\Psi^\dagger(\mathbf{r})\Psi^\dagger(\mathbf{r}')V_2\Psi(\mathbf{r})\Psi(\mathbf{r}') + \cdots$$

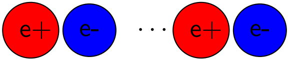
From Lattice QFT to EFT: The Schwinger Model

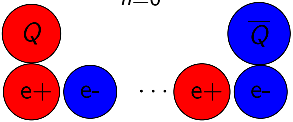
- The Schwinger model is QED in one space and one time dimension
- The Lagrangian for the continuum model:

$$\mathcal{L} \;=\; \overline{\psi}\,(i\not{D} - m)\,\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

- spatially discretized with the Kogut-Susskind (staggered) action
- mapped onto qubits using the JW transformation

$$\hat{H} = x\sum_{n=0}^{N_Q-1}\left(\sigma_n^+ L_n^- \sigma_{n+1}^- + \sigma_{n+1}^+ L_n^+ \sigma_n^-\right) + \sum_{n=0}^{N_Q-1}\left(l_n^2 + \frac{\mu}{2}(-)^n Z_n\right)$$





How Can Near-Term Quantum Computers Help in Scientific Applications?

- Calculating N-qubit observables on a classical computer is a problem for $N \geq 50$
  - $|\psi\rangle$ is an N-qubit state, $\hat{O}$ is an N-qubit observable
  - Representing $|\psi\rangle$ needs $O(2^N)$ classical bits
  - $\langle\psi|\hat{O}|\psi\rangle$ reduces to a matrix-vector multiplication on a classical computer with $O(2^{2N})$ operations
- Calculating N-qubit observables on a quantum computer requires N qubits
  - Prepare the N-qubit state $|\psi\rangle$ M times
  - Measure $\hat{O}$ each time $|\psi\rangle$ is prepared
  - Compute the average $\approx \langle\psi|\hat{O}|\psi\rangle$

Particularly of interest in scientific apps is a ground-state energy of N-body systems.

Algorithm 1 Variational Quantum Eigensolver Algorithm

initialize variational parameter $\boldsymbol{\theta}_0 = \{\theta_0^1, \cdots, \theta_0^M\}$

while $\mathrm{grad}(\langle\Psi(\boldsymbol{\theta_i})|H|\Psi(\boldsymbol{\theta_i})\rangle) \geq \epsilon$ **do**

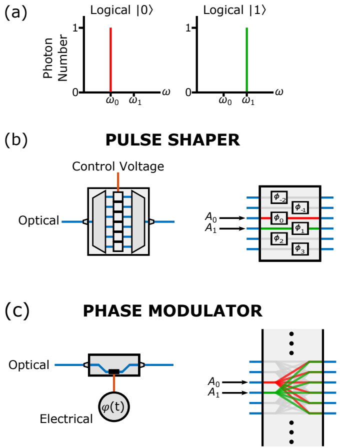
quantum compute $\langle\Psi(\boldsymbol{\theta}_i)|H|\Psi(\boldsymbol{\theta}_i)\rangle$

classical update $\boldsymbol{\theta}_{i+1}$

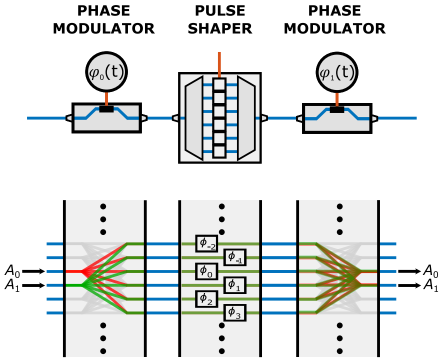
return min $\langle\Psi(\boldsymbol{\theta})|H|\Psi(\boldsymbol{\theta})\rangle$

Introducing the Hardware: Quantum Frequency Processor

Building Blocks



Quantum Frequency Processor



Please see the talk by Joe Lukens on Friday

Optica **4** , 8-16 (2017)

PRL **120**, 030502 (2018)

Optica **5** , 1455-1460 (2018)

Mapping Subatomic Problems onto a QFP

- Start with a N-body second-quantized Hamiltonian H_{SQ}
- Project H_{SQ} onto eigenstates of operators that represent good quantum numbers (e.g., parity, momentum, total spin) for the system of interest. ($\tilde{\mathcal{H}}_{SQ} = \bigoplus_i \mathcal{H}_i$)
- $\mathcal{H}_i$s can now be interpreted as single-particle Hamiltonians and mapped onto a frequency bin multiport device
- $$\mathcal{H}^i_{QFP} = \sum_{k=0}^{d-1} h_{kk} c_k^\dagger c_k + \sum_{\substack{k,l=0 \\ k<l}}^{d-1} [h_{kl} c_k^\dagger c_l + h_{kl}^* c_l^\dagger c_k]$$
- Single particle (photon) variation wavefunction

$$|\Psi\rangle = \cos\phi\,|10\cdots0\rangle - \frac{\sin\phi}{\phi}\sum_{k=1}^{d-1}\theta_k|0\cdots1_k\cdots0\rangle$$

VQE: Single Photon vs Coherent State

- We use QFP to compute $H_{k,l} = \langle\Psi|h_{kl}c_k^\dagger c_l + h_{kl}^* c_l^\dagger c_k|\Psi\rangle$ for all k, l
- For $H_{k,l}$ computation single-photon state

$$|\Psi\rangle = \cos\phi\,|10\cdots0\rangle - \frac{\sin\phi}{\phi}\sum_{k=1}^{d-1}\theta_k|0\cdots1_k\cdots0\rangle$$

and

$$|\Psi_{\mathrm{comb}}\rangle = |\alpha\cos\phi\rangle \otimes \left|\alpha e^{i\pi}\frac{\theta_1\sin\phi}{\phi}\right\rangle \otimes\cdots\otimes \left|\alpha e^{i\pi}\frac{\theta_{d-1}\sin\phi}{\phi}\right\rangle$$

are equivalent

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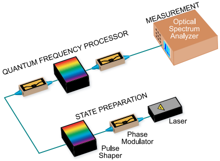
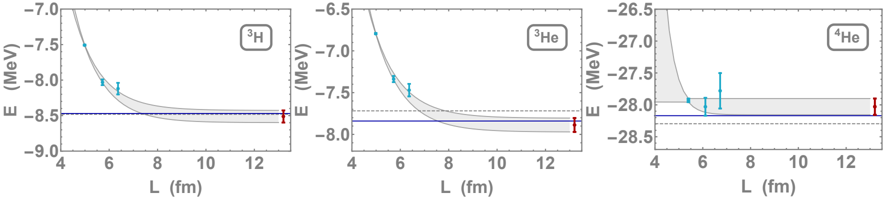
Light Nuclei Simulations

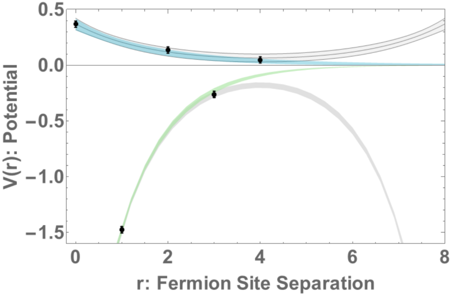


All-optical quantum frequency processor



[arXiv:1810.03959]

EFT for the Schwinger 4-lattice-site Problem



Effective 2-body potential between heavy charges in the 4-lattice-site Schwinger model

[arXiv:1810.03959]

Acknowledgments

Natalie Klco, Martin Savage, Alessandro Roggero, Kody Law, Ajay Jasra, Mikel Sanz, Lucas Lamata, Enrique Solano, Ryan Bennink, Travis Humble, Thomas Maier, Alex McCaskey, Shirley Moore, Nicholas Peters, Gaute Hagen, Gustav Jansen, David Dean, Raphael Pooser, Eugene Dumitrescu, Hsuan-Hao Lu, Joe Lukens, Aaina Bansal, Andreas Ekström, Andrew M Weiner, Titus Morris, Thomas Papenbrock.

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ORNL LDRD program

Thank You! Questions?

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Multiple Postdoc Positions

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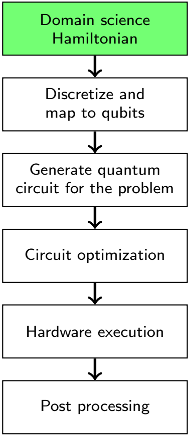
Quantum Simulation Algorithms: A General Workflow

- Typically an observable of interest is energy
- Take a Hamiltonian $H = T + V$ and write it in a second-quantized form using field operators $\Psi(\mathbf{r})$, $\Psi^\dagger(\mathbf{r})$
- The kinetic energy $T \to \int d\mathbf{r} \Psi^\dagger(\mathbf{r}) T \Psi(\mathbf{r})$
- The potential energy

$$V \quad \to \quad \int d\mathbf{r} \Psi^\dagger(\mathbf{r}) V_1 \Psi(\mathbf{r}) + \int d\mathbf{r} d\mathbf{r}' \Psi^\dagger(\mathbf{r}) \Psi^\dagger(\mathbf{r}') V_2 \Psi(\mathbf{r}) \Psi(\mathbf{r}') + \cdots$$

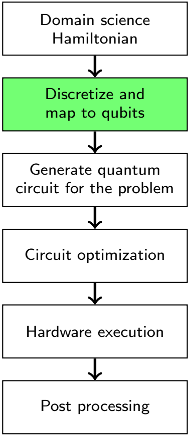


Quantum Simulation Algorithms: A General Workflow

- Discretize field operators $\Psi(\mathbf{r}) = \sum_{n=0}^{N} \psi_n(\mathbf{r}) c_n$, c_n is a single-particle annihilation operator
- Re-write H in terms of c_n and $c_n^\dagger$:

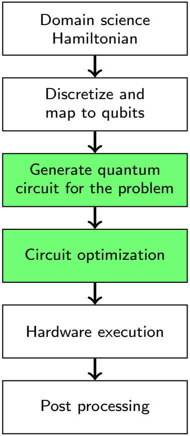
$$H = \sum_{n,m} (T_{nm} + V_{nm}) c_n^\dagger c_m + \sum_{n,m,k,l} V_{nmkl} c_n^\dagger c_m^\dagger c_k c_l + \cdots$$

- Map c_n and $c_n^\dagger$ onto Pauli X, Y, Z operators (qubits = spin 1/2!).



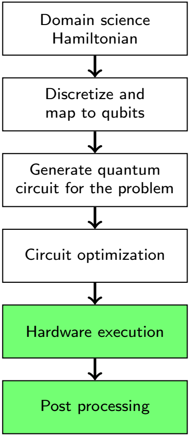
Quantum Simulation Algorithms: A General Workflow

- We will need to prep an N-qubit state $|\psi(\theta_1, \cdots \theta_N)\rangle$ to evaluate $\langle\psi(\theta_1, \cdots \theta_N)|H|\psi(\theta_1, \cdots \theta_N)\rangle$
- $|\psi(\theta_1, \cdots \theta_N)\rangle = U(\theta_1, \cdots \theta_N)|0, \cdots, 0\rangle$
- We will need to decompose $U(\theta_1, \cdots \theta_N)$ into a sequence of one- and two-qubit operations that quantum hardware can implement
- Optimize the decomposition to minimize the use of precious two-qubit gates

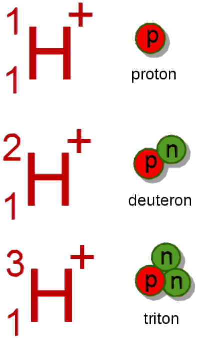


Quantum Simulation Algorithms: A General Workflow

- Existing quantum hardware is noisy. Simulation results require post processing to remove systematic effects of noise



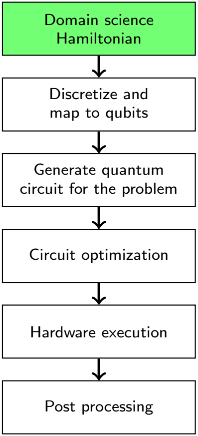
An Example: The Deuteron



- The deuteron is the lightest nucleus consisting of a neutron and a proton
- Binding energy $E_B = -2.2$ MeV (measured)
- Use quantum computer to calculate the binding energy

The Deuteron: Hamiltonian Model

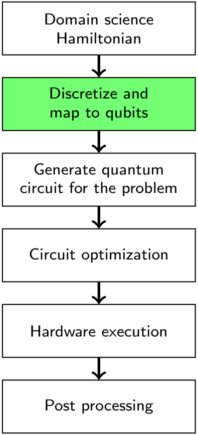
- The Deuteron is a two-body system with the Hamiltonian in the center-of-mass frame $H = T + V$
- The kinetic energy $T = \frac{p^2}{2\mu}$
- The potential $V(r)$ is specially chosen to reproduce deuteron's binding energy
- We want to use quantum computers to calculate the smallest eigenvalue of H (binding energy)
- More precisely, we would like to compute $\langle \Psi | H | \Psi \rangle$ for a class of $|\Psi\rangle$
- Minimizing $\langle \Psi | H | \Psi \rangle$ over all $|\Psi\rangle$ will give an estimate of the binding energy



The Deuteron: Second-Quantized Hamiltonian

- First we need to rewrite $H = \frac{p^2}{2\mu} + V(r)$ in the second-quantized form
- We use harmonic oscillator basis (Fock states $|n\rangle, n = 0, \cdots, N$ and single particle creation (annihilation) operators $c_n^\dagger (c_n)$)
- In this basis $H_N = \sum_{n,n'=0}^{N-1} \langle n'|H|n\rangle c_{n'}^\dagger c_n$
- We evaluate matrix elements in the Fock basis:

$$\begin{aligned}
\langle n'|V(r)|n\rangle &= V_0 \delta_n^0 \delta_n^{n'} \\
\langle n'|\frac{p^2}{2\mu}|n\rangle &= \frac{\hbar\omega}{2}\left[(2n+3/2)\delta_n^{n'} - \sqrt{n(n+1/2)}\delta_n^{n'+1}\right. \\
&\quad \left. -\sqrt{(n+1)(n+3/2)}\delta_n^{n'-1}\right]
\end{aligned}$$



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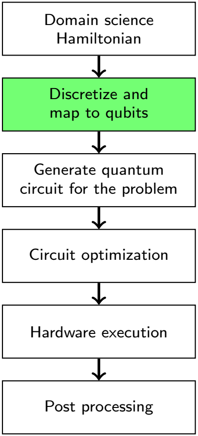
The Deuteron: Mapping to Qubits

- We use Jordan-Wigner transform

$$c_n^\dagger \to \frac{1}{2}\left[\prod_{j=0}^{n-1} -Z_j\right](X_n - iY_n);\; c_n \to \frac{1}{2}\left[\prod_{j=0}^{n-1} -Z_j\right](X_n + iY_n)$$

- A spin up $|\uparrow\rangle$ (down $|\downarrow\rangle$) state of the qubit n corresponds to zero (one) deuteron in the state $|n\rangle$.
- Hamiltonians for $N = 2, 3$:

$$\begin{aligned} H_2 &= 5.906709 I + 0.218291 Z_0 - 6.125 Z_1 \\ &\quad - 2.143304\,(X_0X_1 + Y_0Y_1)\,, \\ H_3 &= H_2 + 9.625(I - Z_2) \\ &\quad - 3.913119\,(X_1X_2 + Y_1Y_2) \end{aligned}$$



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The Deuteron: Computing the Binding Energy

- Compute the binding energy E_N using VQE and the Unitary Coupled-Cluster ansatz for all N up to N_{max}
- Extrapolate to the continuum E_∞ by using $E_1, \cdots, E_{N_{max}}$
- For the extrapolation to the infinite space we employ the harmonic-oscillator variant of Lüscher's formula for finite-size corrections to the ground-state energy:

$$\begin{aligned} E_N &= -\frac{\hbar^2 k^2}{2m}\left(1 - 2\frac{\gamma^2}{k}e^{-2kL} - 4\frac{\gamma^4 L}{k}e^{-4kL}\right) \\ &\quad + \frac{\hbar^2 k \gamma^2}{m}\left(1 - \frac{\gamma^2}{k} - \frac{\gamma^4}{4k^2} + 2w_2 k\gamma^4\right)e^{-4kL}, \end{aligned}$$

$L = L(N)$ is the effective hard-wall radius for the finite basis of dimension N, k is the bound-state momentum, γ the asymptotic normalization coefficient, and w_2 an effective range parameter.

The Deuteron: VQE and UCC Quantum Circuits

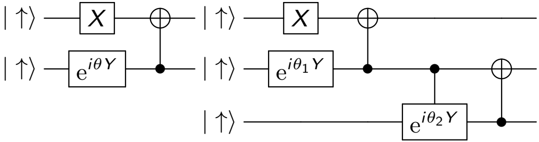
- Unitary Coupled-Cluster ansatz for $N = 2$ (two qubits):

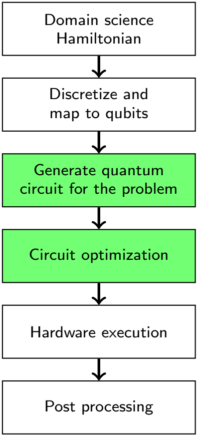
$$|\Psi_{UCC}\rangle \equiv e^{\theta\left(c_0^\dagger c_1 - c_1^\dagger c_0\right)}|\downarrow\uparrow\rangle = \cos\theta|\downarrow\uparrow\rangle - \sin\theta|\uparrow\downarrow\rangle$$

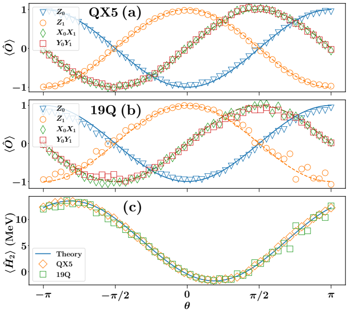
- Unitary Coupled-Cluster ansatz for $N = 3$ (three qubits):

$$\begin{aligned} |\Psi_{UCC}\rangle &\equiv e^{\sum_{i=1}^{N-1}\theta_i\left(c_0^\dagger c_i - c_i^\dagger c_0\right)}|\downarrow\uparrow\uparrow\rangle = \cos(\theta_1)|\downarrow\uparrow\uparrow\rangle \\ &- \sin(\theta_1)\cos(\theta_2)|\uparrow\downarrow\uparrow\rangle - \sin(\theta_1)\sin(\theta_2)|\uparrow\uparrow\downarrow\rangle \end{aligned}$$

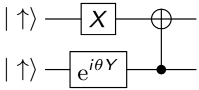
- Quantum circuits:





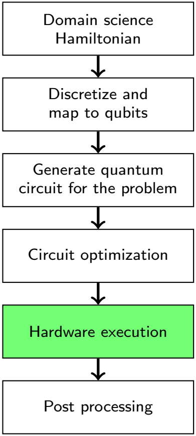


- Quantum circuit

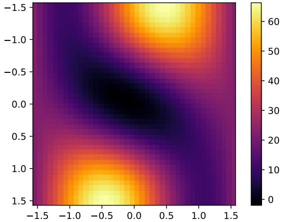


- Hamiltonian:

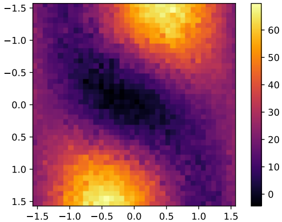
$$H_2 = 5.906709 I + 0.218291 Z_0 - 6.125 Z_1 - 2.143304 \left(X_0 X_1 + Y_0 Y_1 \right)$$



The Deuteron: VQE Experimental Results $N = 3$



Simulated $E_3(\theta_1, \theta_2)$

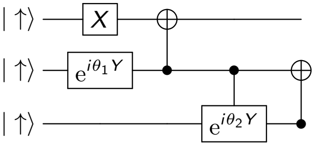


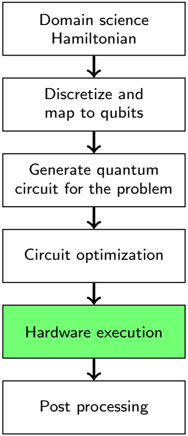
Measured $E_3(\theta_1, \theta_2)$ on IBM QX 5

- Hamiltonian:

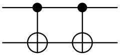
$$H_3 = H_2 + 9.625(I - Z_2) - 3.913119\,(X_1 X_2 + Y_1 Y_2)$$

- Quantum circuit





- Two-qubit CNOT gates are imperfect, with error rate e

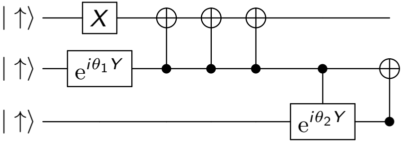


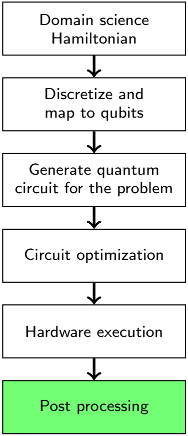
$$= \mathbb{1} + 2e\mathcal{E}$$

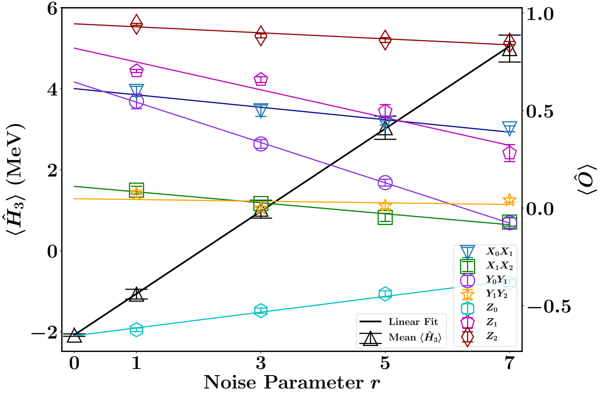
- If e is small, inserting noisy $\mathbb{1}$ gates into the variational circuit will result in the energy change

$$H(r) = H(0) - reH(0)$$

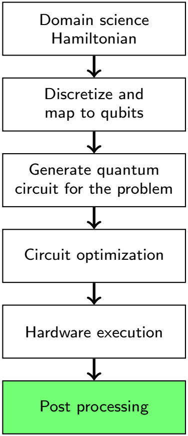
- It is a linear regression $\{r, H(r)\}$!







Zero-noise extrapolation around the minimum



The Deuteron: Putting It All Together

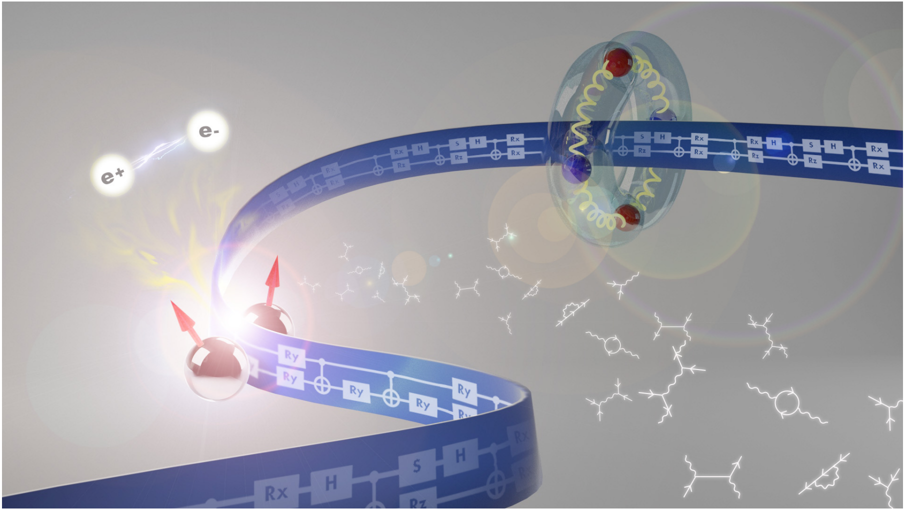
The exact binding energy of the deuteron $E_\infty = -2.22$ MeV.

E from exact diagonalization				
N	E_N	$\mathcal{O}(e^{-2kL})$	$\mathcal{O}(kLe^{-4kL})$	$\mathcal{O}(e^{-4kL})$
2	-1.749	-2.39	-2.19	
3	-2.046	-2.33	-2.20	-2.21
E from quantum computing				
N	E_N	$\mathcal{O}(e^{-2kL})$	$\mathcal{O}(kLe^{-4kL})$	$\mathcal{O}(e^{-4kL})$
2	$-1.74(3)$	$-2.38(4)$	$-2.18(3)$	
3	$-2.08(3)$	$-2.35(2)$	$-2.21(3)$	$-2.28(3)$

$$
\begin{aligned}
E_N &= -\frac{\hbar^2 k^2}{2m}\left(1 - 2\frac{\gamma^2}{k}e^{-2kL} - 4\frac{\gamma^4 L}{k}e^{-4kL}\right) \\
&\quad + \frac{\hbar^2 k\gamma^2}{m}\left(1 - \frac{\gamma^2}{k} - \frac{\gamma^4}{4k^2} + 2w_2 k\gamma^4\right)e^{-4kL}
\end{aligned}
$$

[PRL 120 210501; arXiv:1801.03897]

Lattice Quantum Field Theories on a Quantum Computer



The Schwinger Model: A Quick Overview

- The Schwinger model describes quantum electrodynamics in one space and one time dimension, $1+1$
- Key features: confinement and spontaneous breaking of chiral symmetry (QCD-like)
- The vacuum of the theory enjoys a non-zero condensate, $\langle\psi\overline{\psi}\rangle$
- The Lagrangian for the continuum model:

$$\mathcal{L} \;=\; \overline{\psi}\,(i\not{D} - m)\,\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

The Schwinger Model: Discretization

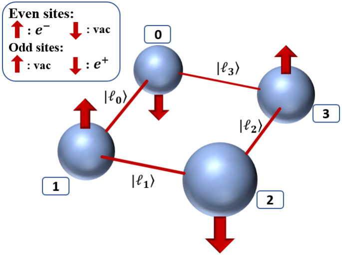
The Schwinger model can be

- spatially discretized with the Kogut-Susskind (staggered) action
- mapped onto a (re-scaled) Hamiltonian density using the Jordan-Wigner transformation
- gauge-fixed by setting the temporal component of the gauge field to zero ($A_0 = 0$) on $N_Q/2$ spatial sites

$$\hat{H} = x \sum_{n=0}^{N_Q-1} \left(\sigma_n^+ L_n^- \sigma_{n+1}^- + \sigma_{n+1}^+ L_n^+ \sigma_n^-\right) + \sum_{n=0}^{N_Q-1} \left(l_n^2 + \frac{\mu}{2}(-)^n Z_n\right)$$

[we will use $x = 0.6$ and $\mu = 0.1$ throughout this presentation]

The Schwinger Model: Two Spatial Lattice Sites



Qubit and electric flux link structure of the two-spatial-site lattice Schwinger model

Naïve qubit count

- Two qubits are sufficient to describe the fermion occupation of a single spatial lattice site, one for the e^- and one the e^+.
- We cut off the range of values of each l_n ($|l_n| \leq 1$), thus two qubits per flux link
- Total qubit # to simulate two-lattice site dynamics $2 + 2 + 4 * 2 = \mathbf{12}$

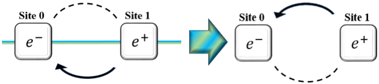
The Schwinger Model: Making Use of the Symmetries

Natural symmetries of the Schwinger $1+1$ model:

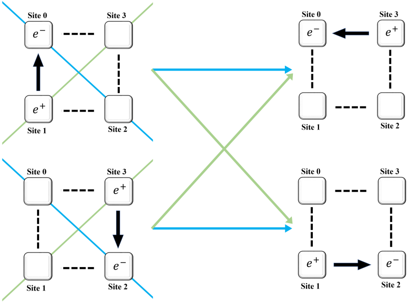
- Local Abelian gauge symmetry $U(1)$
- *Parity* transformation $\hat{P}_a$ (reflection of the system through axes that preserves the structure of the Wigner-Jordan representation of the fermionic fields)
- *Charge conjugation* $\hat{C}_i$ transforms particles into antiparticles and vice versa, and the direction of the electric field reverses as a result (an additional directional shift by one lattice site is necessary)
- *Translation* symmetry (natural on a lattice with PBC) leads to the momentum conservations

The Schwinger Model: Parity Symmetry Examples

Parity transform for the one-spatial-site lattice Schwinger mode:



Parity transform for the two-spatial-site lattice Schwinger model:



The Schwinger Model: Physical States

13 physical states satisfy Gauss's law in the charge $Q = 0$ sector:

$$|\phi_1\rangle = |\cdot\cdot\cdot\cdot\rangle|0000\rangle$$
$$|\phi_2\rangle = |\cdot\cdot\cdot\cdot\rangle|1111\rangle$$
$$|\phi_3\rangle = |\cdot\cdot\cdot\cdot\rangle|-1-1-1-1\rangle$$
$$|\phi_4\rangle = |e^-e^+\cdot\cdot\rangle|-1000\rangle$$
$$|\phi_5\rangle = |\cdot\cdot e^-e^+\rangle|00-10\rangle$$
$$|\phi_6\rangle = |e^-e^+\cdot\cdot\rangle|0111\rangle$$
$$|\phi_7\rangle = |\cdot\cdot e^-e^+\rangle|1101\rangle$$

$$|\phi_8\rangle = |e^-e^+e^-e^+\rangle|-10-10\rangle$$
$$|\phi_9\rangle = |e^-e^+e^-e^+\rangle|0101\rangle$$
$$|\phi_{10}\rangle = |e^-\cdot\cdot e^+\rangle|-1-1-10\rangle$$
$$|\phi_{11}\rangle = |e^-\cdot\cdot e^+\rangle|0001\rangle$$
$$|\phi_{12}\rangle = |\cdot e^+e^-\cdot\rangle|0100\rangle$$
$$|\phi_{13}\rangle = |\cdot e^+e^-\cdot\rangle|-10-1-1\rangle$$

The Schwinger Model: Translation Invariant States

Sectors of definite momentum, $\mathbf{k}$, constrained to satisfy $\mathbf{k} = \pi n$ with $n = 0, \pm 1$:

$$|\psi_1\rangle_{\mathbf{k}=\mathbf{0}} = |\phi_1\rangle$$
$$|\psi_2\rangle_{\mathbf{k}=\mathbf{0}} = |\phi_2\rangle$$
$$|\psi_3\rangle_{\mathbf{k}=\mathbf{0}} = |\phi_3\rangle$$
$$|\psi_4\rangle_{\mathbf{k}=\mathbf{0}} = \frac{1}{\sqrt{2}}\ [\ |\phi_4\rangle + |\phi_5\rangle\ ]$$
$$|\psi_5\rangle_{\mathbf{k}=\mathbf{0}} = \frac{1}{\sqrt{2}}\ [\ |\phi_6\rangle + |\phi_7\rangle\ ]$$

$$|\psi_6\rangle_{\mathbf{k}=\mathbf{0}} = |\phi_8\rangle$$
$$|\psi_7\rangle_{\mathbf{k}=\mathbf{0}} = |\phi_9\rangle$$
$$|\psi_8\rangle_{\mathbf{k}=\mathbf{0}} = \frac{1}{\sqrt{2}}\ [\ |\phi_{10}\rangle + |\phi_{13}\rangle\ ]$$
$$|\psi_9\rangle_{\mathbf{k}=\mathbf{0}} = \frac{1}{\sqrt{2}}\ [\ |\phi_{11}\rangle + |\phi_{12}\rangle\ ]$$

The Schwinger Model: Gauge, Momentum, Charge & Parity

Sectors of definite momentum, $\mathbf{k} = \mathbf{0}$, $\hat{C}\hat{P} = \pm 1$, and charge $Q = 0$:

$$|\chi_1\rangle_{\mathbf{k}=\mathbf{0},+} = |\psi_1\rangle$$

$$|\chi_2\rangle_{\mathbf{k}=\mathbf{0},+} = \frac{1}{\sqrt{2}} \left[\, |\psi_4\rangle + |\psi_9\rangle \,\right]$$

$$|\chi_3\rangle_{\mathbf{k}=\mathbf{0},+} = \frac{1}{\sqrt{2}} \left[\, |\psi_6\rangle + |\psi_7\rangle \,\right]$$

$$|\chi_4\rangle_{\mathbf{k}=\mathbf{0},+} = \frac{1}{\sqrt{2}} \left[\, |\psi_5\rangle + |\psi_8\rangle \,\right]$$

$$|\chi_5\rangle_{\mathbf{k}=\mathbf{0},+} = \frac{1}{\sqrt{2}} \left[\, |\psi_2\rangle + |\psi_3\rangle \,\right]$$

$$|\chi_1\rangle_{\mathbf{k}=\mathbf{0},-} = \frac{1}{\sqrt{2}} \left[\, |\psi_4\rangle - |\psi_9\rangle \,\right]$$

$$|\chi_2\rangle_{\mathbf{k}=\mathbf{0},-} = \frac{1}{\sqrt{2}} \left[\, |\psi_6\rangle - |\psi_7\rangle \,\right]$$

$$|\chi_3\rangle_{\mathbf{k}=\mathbf{0},-} = \frac{1}{\sqrt{2}} \left[\, |\psi_5\rangle - |\psi_8\rangle \,\right]$$

$$|\chi_4\rangle_{\mathbf{k}=\mathbf{0},-} = \frac{1}{\sqrt{2}} \left[\, |\psi_2\rangle - |\psi_3\rangle \,\right]$$

The Schwinger Model: Recapping the Symmetries

- By using the eigenstates of the Momentum, Charge, Parity operators we have preconditioned the Schwinger Hamiltonian H,

$$H \equiv \bigoplus_{i=\{\mathbf{k},CP\}} H_i$$

- The non-trivial dynamics happens in two subspaces:

$$\mathrm{H}_{\mathbf{k=0},+} = \begin{pmatrix} -2\mu & 2x & 0 & 0 & 0 \\ 2x & 1 & \sqrt{2}x & 0 & 0 \\ 0 & \sqrt{2}x & 2+2\mu & \sqrt{2}x & 0 \\ 0 & 0 & \sqrt{2}x & 3 & \sqrt{2}x \\ 0 & 0 & 0 & \sqrt{2}x & 4-2\mu \end{pmatrix}$$

$$\mathrm{H}_{\mathbf{k=0},-} = \begin{pmatrix} 1 & \sqrt{2}x & 0 & 0 \\ \sqrt{2}x & 2+2\mu & -\sqrt{2}x & 0 \\ 0 & -\sqrt{2}x & 3 & \sqrt{2}x \\ 0 & 0 & \sqrt{2}x & 4-2\mu \end{pmatrix}$$

- Reduced # of qubits from **12** to $\leq$**3**!

The Schwinger Model: Dynamics Simulations

- Working with the $\mathbf{k}=\mathbf{0}$ $CP=+1$ sector, we are interested in the dynamics of the unoccupied state $|\chi_1\rangle_{\mathbf{k}=\mathbf{0},+}$

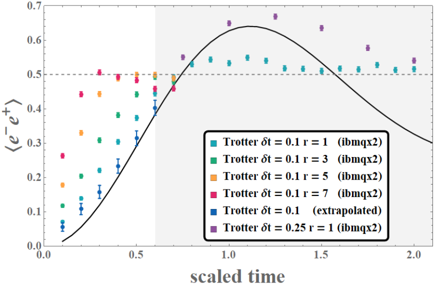
- Trotterization

$$e^{-iHt} \to \lim_{N\to\infty} \left(\prod_j e^{-iH_j \delta t} \right)^N$$

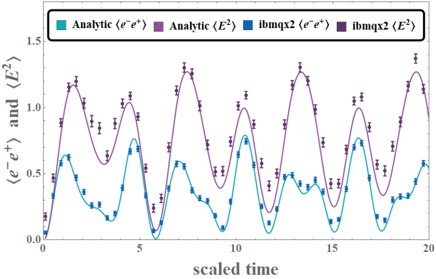
- Exact $SU(4)$ dynamics
  $e^{-iHt} = K^T C K$ where
  $C \approx e^{-iXX} e^{-iYY} e^{-iZZ}$
  $K \in SU(2) \otimes SU(2)$

The Schwinger Model: Simulation Results IBM QX2

- Trotterization



- Exact $SU(4)$ dynamics



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