Efficiently Certifying Large Amounts of Entanglement in High-Dimensional Quantum Systems

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Photonics for Quantum Workshop
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Big Picture

Logical Layer

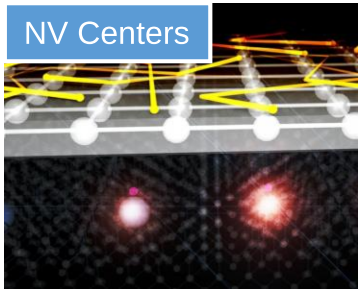
Sensing Protocols (e.g. Cramer-Rao Bound/Quantum Limit)

Algorithms (e.g. Shor/Grover)

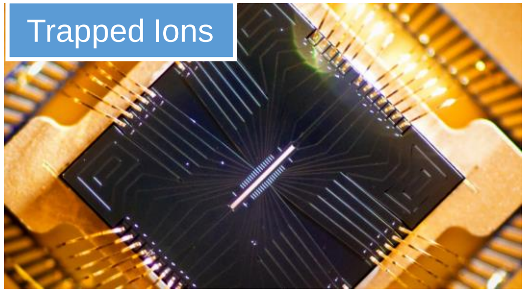
Secure Communication Protocols

Middle “Interface” Layer

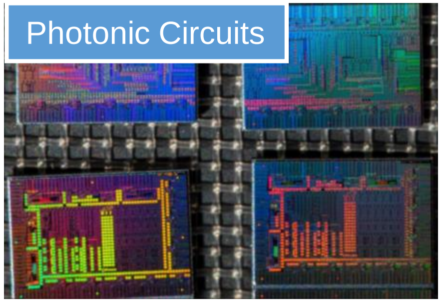
Platform Layer



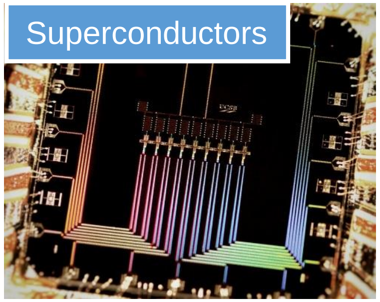
https://jqi.umd.edu/news/quantum-sensor-nanoscale-electron-transport



https://jqi.umd.edu/news/future-ion-traps



AFRL/RIT



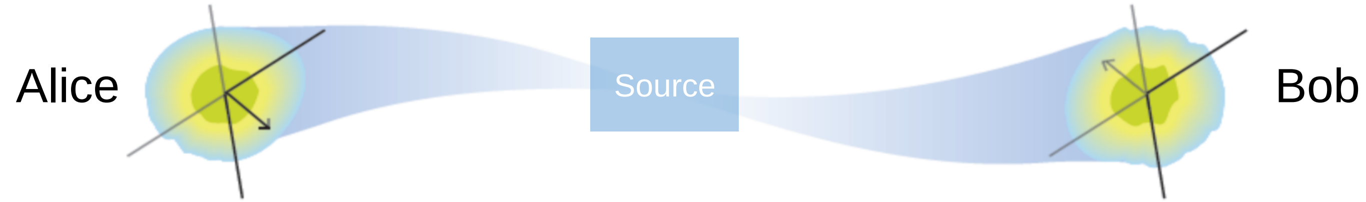
http://web.physics.ucsb.edu/~martinisgroup/

Big Picture

Platforms are varied and imperfect.

We need practical tools for characterizing large-scale quantum systems and the resources they provide.

Quantify Entanglement in High Dimensions



What do I mean by high-dimensions?

- Quantum system divided into two parts (for now)
- Each party has n local measurement outcomes where n is big (100,1000,10000,512^2……)
- Measuring a joint-distribution is n^2 measurements
- Hilbert space can be continuous or discrete

Examples

- Photon pairs entangled in spatial variables (position/momentum, orbital angular momentum)
- Photons entangled in energy-time (or time bins/frequency bins)
- Path entangled qudits on chip
- Divide atomic/superconducting systems in two (may be separate, may not be)

Quantifying Entanglement Overview

- Quantifying Entanglement
  - Most general way is to compute an *entanglement monotone*
    - **Entanglement of Formation**: The number of two-qubit Bell pairs required on average to synthesize the desired quantum state
    - **Distillable Entanglement**: The number of two-qubit Bell pairs on average that can be synthesized from a given quantum state
    - **Measured in units of *ebits.* 1 *ebit* is the quantity of entanglement in a perfect, two qubit Bell State.**
    - Computing these requires the density matrix which is difficult to acquire at large dimension
  - Entanglement-Dimensionality (Schmidt Rank)
  - Usually we can only *witness* entanglement in large quantum systems or use application-specific metrics
- Quantitative Entanglement Witnesses
  - **lower-bounds** for entanglement monotones that can be computed from limited data (e.g. do not require a density matrix)

Quantifying Entanglement

The Hard Way

- Perform a full experimental unbiased quantum tomography
  - Scales exponentially with added particles and at least quadratically with added dimensions
- Compute an entanglement monotone (e.g. ebits)
  - NP-Hard in general

The Easy Way?

- Invent a quantitative entanglement witness that…

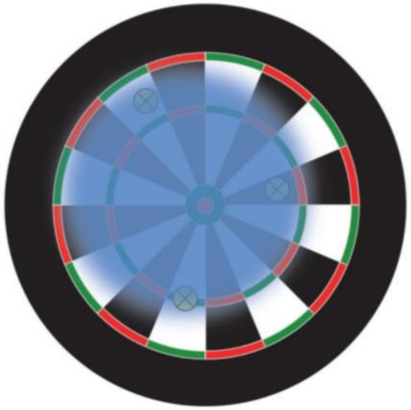
1. Works with very limited data
2. Measurements are easy and “smart”
3. Results not contingent on any assumptions
4. Easy error analysis
5. Avoid numerical inversion/optimization

Uncertainty Relations

Variance-Based

$$\sigma_x \sigma_k \geq 1/2$$

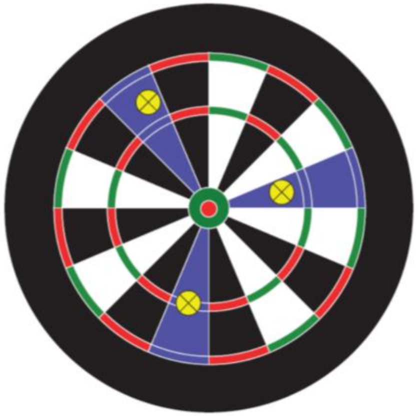
- Variance captures localization or clustering



Information-Based (Entropic)

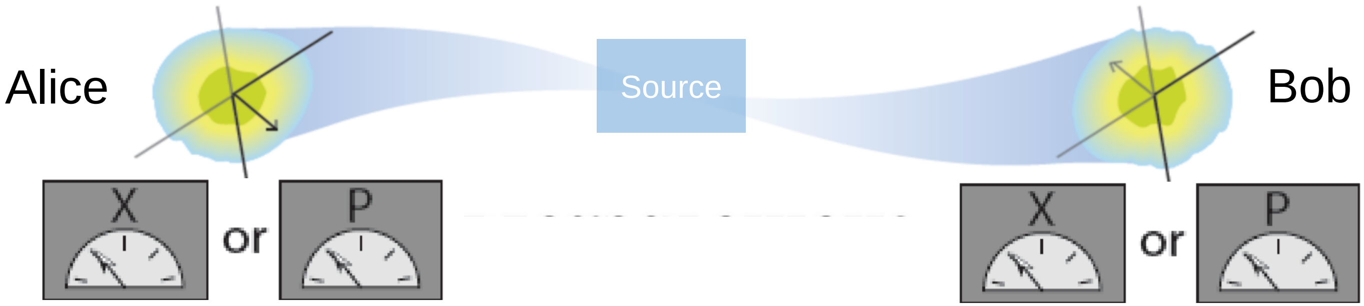
$$h(x) + h(k) \geq \pi e$$

- h is Shannon Entropy (bits)
- Entropy captures the number of questions you have to ask? (1 bit = 1 yes/no question)



The more information a measurement tells you about position statistics, the less information a subsequent measurement can tell you about momentum statistics.

Witnessing the EPR Paradox



EPR Paradox is seen when $\sigma_{x_A|x_B}\sigma_{k_A|k_B} \leq 1/2$

Entropic Witness of the EPR Paradox

$$d \log\left(\frac{\pi e}{\Delta_x \Delta_k}\right) - [H(\boldsymbol{X}_A|\boldsymbol{X}_B) + H(\boldsymbol{K}_A|\boldsymbol{K}_B)] \leq R_{secret}$$

secret key rate

- $H(\boldsymbol{X}_A|\boldsymbol{X}_B)$ is the conditional Shannon Entropy
- (Small when knowledge about system “A” specifies system “B)
- Only requires measuring $P(\boldsymbol{X}_A, \boldsymbol{X}_B)$ and $P(\boldsymbol{K}_A, \boldsymbol{K}_B)$

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Violation of Continuous-Variable Einstein-Podolsky-Rosen Steering with Discrete Measurements

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EPR to Ebits

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Quantifying high-dimensional entanglement with Einstein-Podolsky-Rosen correlations

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(Received 12 September 2017; published 24 April 2018)

$$d\log\left(\frac{2\pi}{\Delta_x\Delta_k}\right) - [H(\boldsymbol{X}_A|\boldsymbol{X}_B) + H(\boldsymbol{K}_A|\boldsymbol{K}_B)] \leq -S(A|B) \leq E_f$$

d --- number of dimensions in vector space
Δ_x--- discretization pixel physical dimension dimension

Need to measure $P(\boldsymbol{X}_A|\boldsymbol{X}_B)$ and $P(\boldsymbol{K}_A|\boldsymbol{K}_B)$
Note: bold indicates vector space observable

$S(A|B)$ is the quantum conditional entropy

Entanglement of Formation (how many ebits you have)

Not doing a tomography anymore, but still a lot of measurements!

Scaling to Many Dimensions

"One can regard the possibility of digital compression as a failure of sensor design. If it is possible to compress measured data, one might argue too many measurements were taken" – David Brady (via EJ Candes)

How can we measure just the "important part" of a signal?

How can we go "Analog to Information"? (DARPA)

Can we replace the "Nyquist Rate" with the "Information Rate"?

Measurements are questions. What are the right questions to ask our system?

Prior Knowledge

- Usually expect some for of structure in a system of interest
- Structure = Compression
- Abstract
  Low rank density matrix, nearly pure state, EPR-type, compressible
- Technical
  Knowledge/model of particular system or hardware

Estimation vs Certification

Estimation is your Best Guess

- Maximum Likelihood Estimation
- **Compressive Sensing**
- Bayesian Inference
- Assumption-based tomography
- Fidelity to a target state
- Numerical/Computational Techniques
- Noise Reduction
- Background Subtraction
- Any of your favorite

Certification is a Guarantee

- For limited measurements, the result is the *worst case* possibility consistent with the measurements
- Essential for quantum security and foundational issues (e.g. closing loopholes)

Can we still certify entanglement if we exploit prior knowledge?

How to exploit *expectations* to improve our Quantitative Entanglement Witness

Assumptions → Estimation

- Result is contingent on assumptions

Expectations → Certification

- Result is valid whether or not expectation is met
- Expectation helps guide you to "asking good questions"
- Outcome will confirm whether you had "great expectations"

$$d \log\left(\frac{2\pi}{\Delta_x \Delta_k}\right) - [H(\boldsymbol{X}_A|\boldsymbol{X}_B) + H(\boldsymbol{K}_A|\boldsymbol{K}_B)] \leq E_f$$

For certification, any approximations we make must **never decrease** the conditional entropy (e.g. decrease the uncertainty)

Good news! When you mess around with conditional entropies you tend to increase them

Improvement 1

Exploit the *expectation* of local separability

A property of conditional entropies is

$$H(\boldsymbol{X_A}|\boldsymbol{X_B}) = H(X_{A1}, \dots, X_{An}|X_{B1}, \dots, X_{Bn}) \leq H(X_{A1}|X_{B1}) + \cdots + H(X_{An}|X_{Bn})$$

Inequality becomes = when $P(\boldsymbol{X}_A, \boldsymbol{X}_B)$ is separable

Example: 2-D Spatial Coordinates

$$\boldsymbol{X} = (X, Y)$$

$$\boldsymbol{then}$$

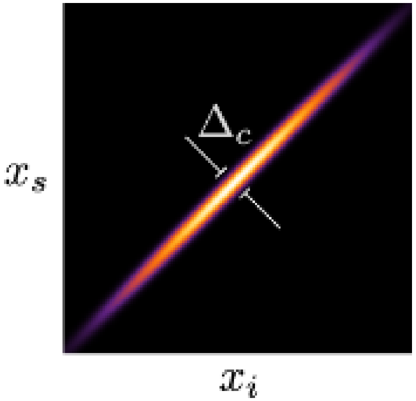
$$H(\boldsymbol{X}_A|\boldsymbol{X}_B) \leq H(X_A|X_b) + H(Y_A|Y_b)$$

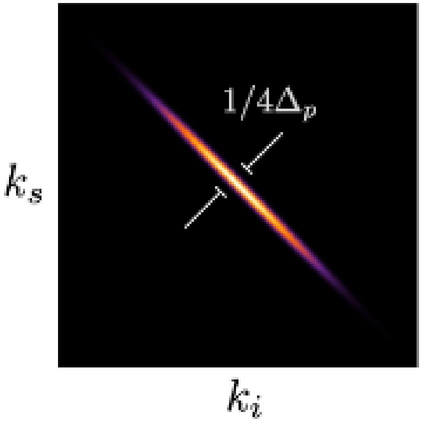
- Instead of measuring $P(\boldsymbol{X_A}, \boldsymbol{X_B})$, measure $P(X_A, X_B)$ and $P(Y_A, Y_B)$ separately $(n^4 \rightarrow 2n^2)$
- Inequality is *always* true even if the distribution isn't actually separable.

Improvement 2

Exploit the *expectation* of Sparsity/Compression

- If our system is highly entangled, we will probably choose to measure observables that are highly correlated
- Highly correlated probability distributions are very sparse
- Want to scan at high-resolution where the correlations are strong and low-resolution elsewhere

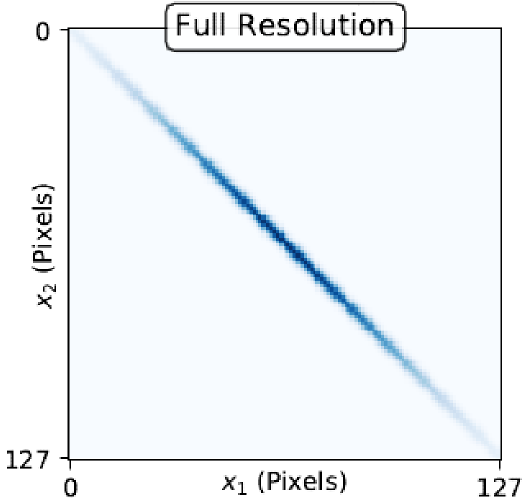




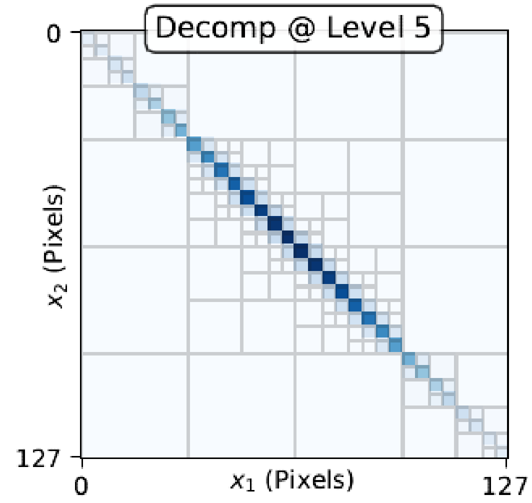
Improvement 2: Multi-Level Approximation

- Replace $P(X_A, X_B)$ with $\tilde{P}(X_A, X_B)$ which is an **arbitrarily** coarse grained estimate of P
- Coarse Graining = replace collections of elements with their average value
- This can only *increase* the conditional entropy (good)
- $H(X_A|X_B) \leq \tilde{H}(X_A|X_B)$ so we can use it!

$P(X_A, X_B)$



$\tilde{P}(X_A, X_B)$



Improved Quantitative Entanglement Witness

In two suggestive dimensions (x,y)…

$$2\log\left(\frac{2\pi}{\Delta_x\Delta_k}\right) - \widetilde{H}(X_A|X_b) + \widetilde{H}(Y_A|Y_b) + \widetilde{H}\left(K_A^{(x)}\middle|K_b^{(x)}\right) + \widetilde{H}\left(K_A^{(Y)}\middle|K_b^{(y)}\right) \leq E_f$$

- Constant that comes from uncertainty relations
- Horizontal Position Correlations (multilevel)
- Vertical Position Correlations (multilevel)
- Horizontal Momentum Correlations (multilevel)
- Vertical Momentum Correlations (multilevel)
- How many ebits we certify

Only assumptions are how probability distributions are found

None are added by the witness

The talk could end here...

Quantifying entanglement in a 68-billion dimensional quantum system

James Schneeloch,[1] Christopher C. Tison,[1,2,3] Michael L. Fanto,[1,4] Paul M. Alsing,[1] and Gregory A. Howland[1,4,*]
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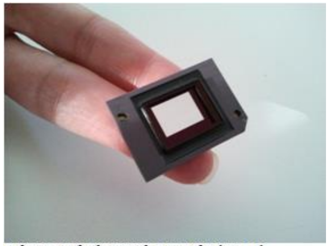
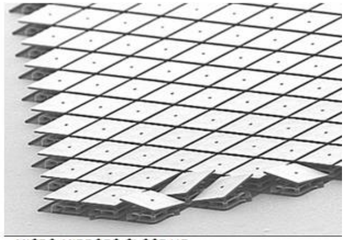
(Dated: May 23, 2018)

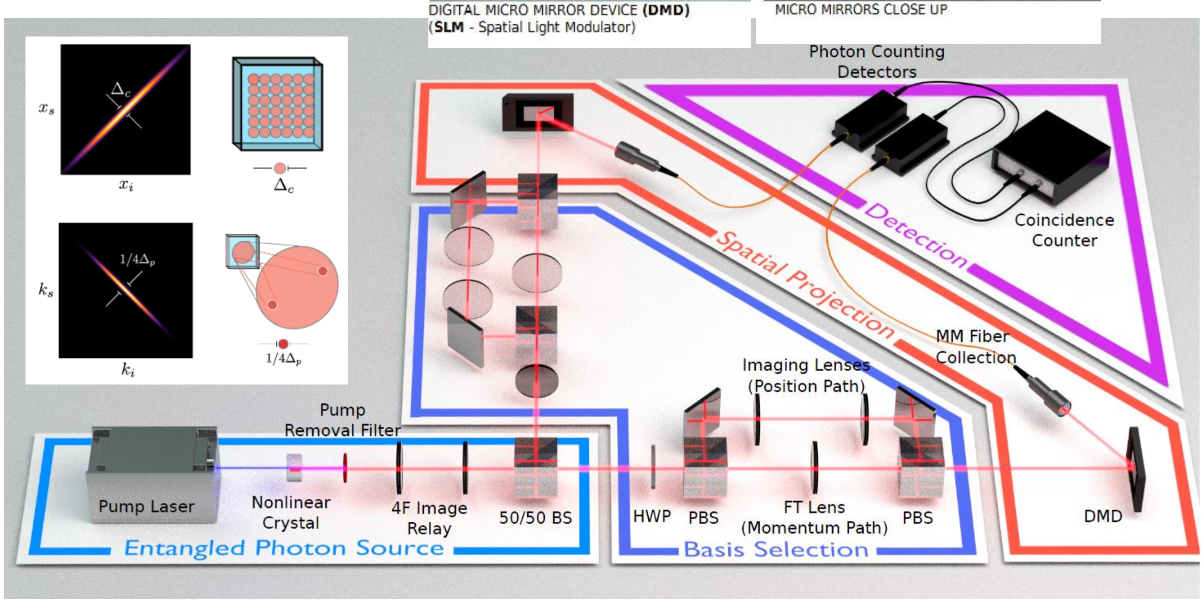
- https://arxiv.org/abs/1804.04515 (in review)
- Quantify 7.1 ebits between spatially entangled photon pairs (512 x 512 pixels per photon)
- Use 20 million times fewer measurements than uncompressed inequality

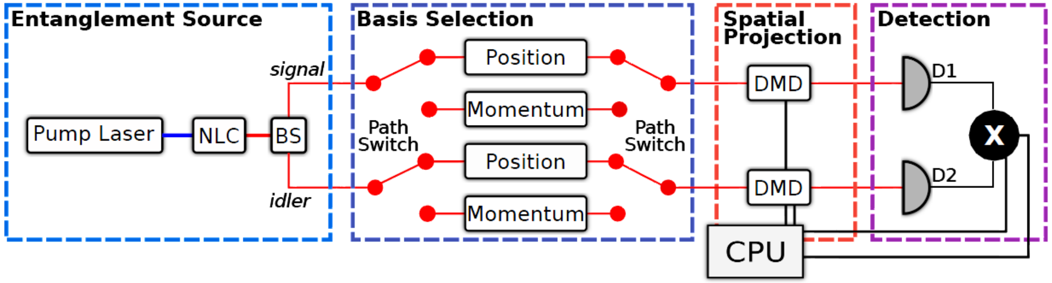
Spatial entanglement testbed

- Easily generated via Spontaneous Parametric Down-Conversion (SPDC) and manipulated with DMDs or SLMs.
- Serves as a proxy-system for energy-time, OAM, other HD bipartite entangled systems.
- Very large entanglement because X and K are continuous variables
- Produces an entangled state very similar to that described in the original EPR paradox.
  - Positive position correlations
  - Negative momentum correlations
- Good for testing new characterization protocols.

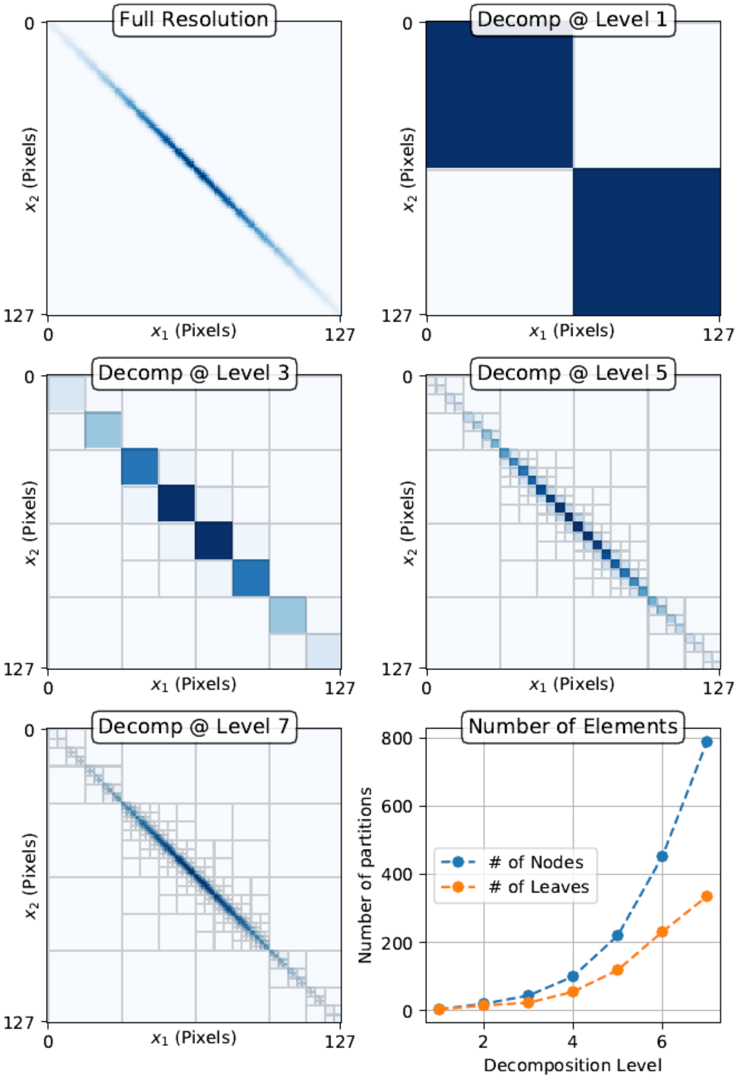






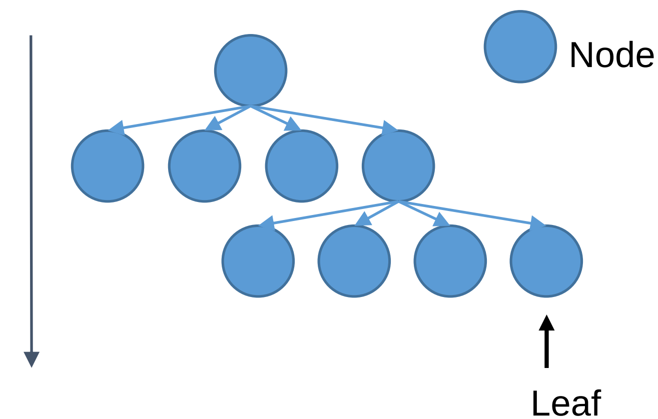


Adaptive, quad-tree sampling

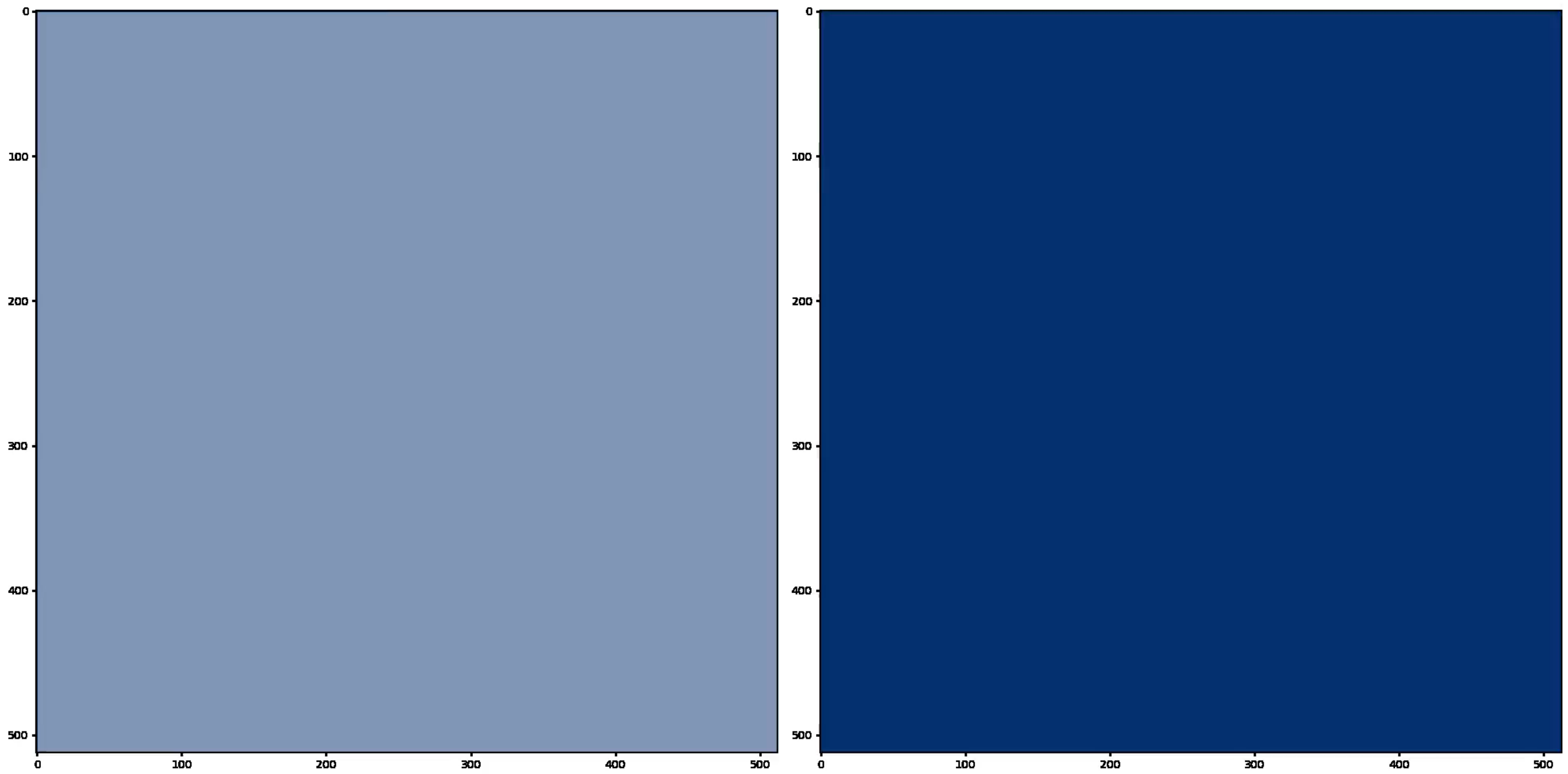


- In an entangled system, we expect distributions of certain observables to be very sparse (e.g. $P(x_1, x_2)$, $P(k_1, k_2)$) due to high intensity correlations
- Want to scan at high-resolution in highly-correlated regions and low-res elsewhere
- Use an adaptive quad-tree sampling technique. Split a block when intensity (coincidence count rate) is greater than a chosen threshold.
- Commonly used in computer graphics for collision detection

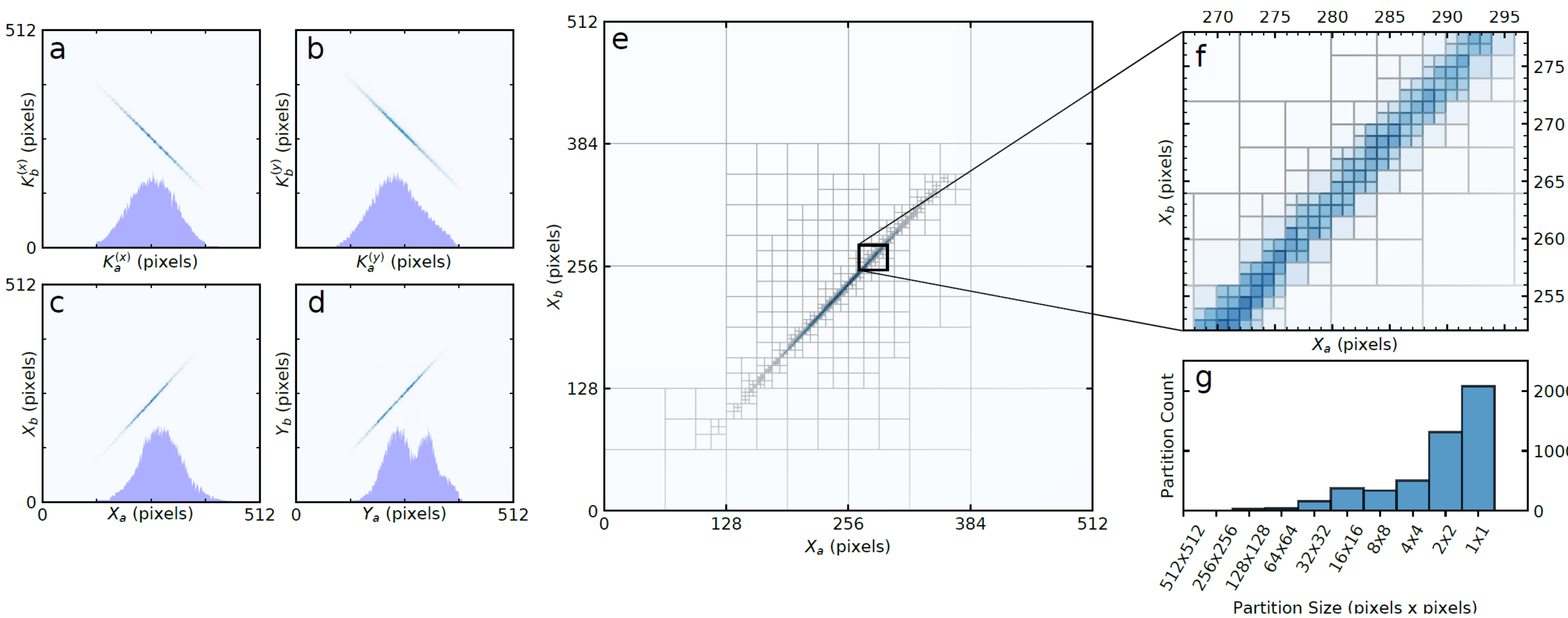
Decreasing pixel size as we go down the tree.



Adaptive Sampling Demo



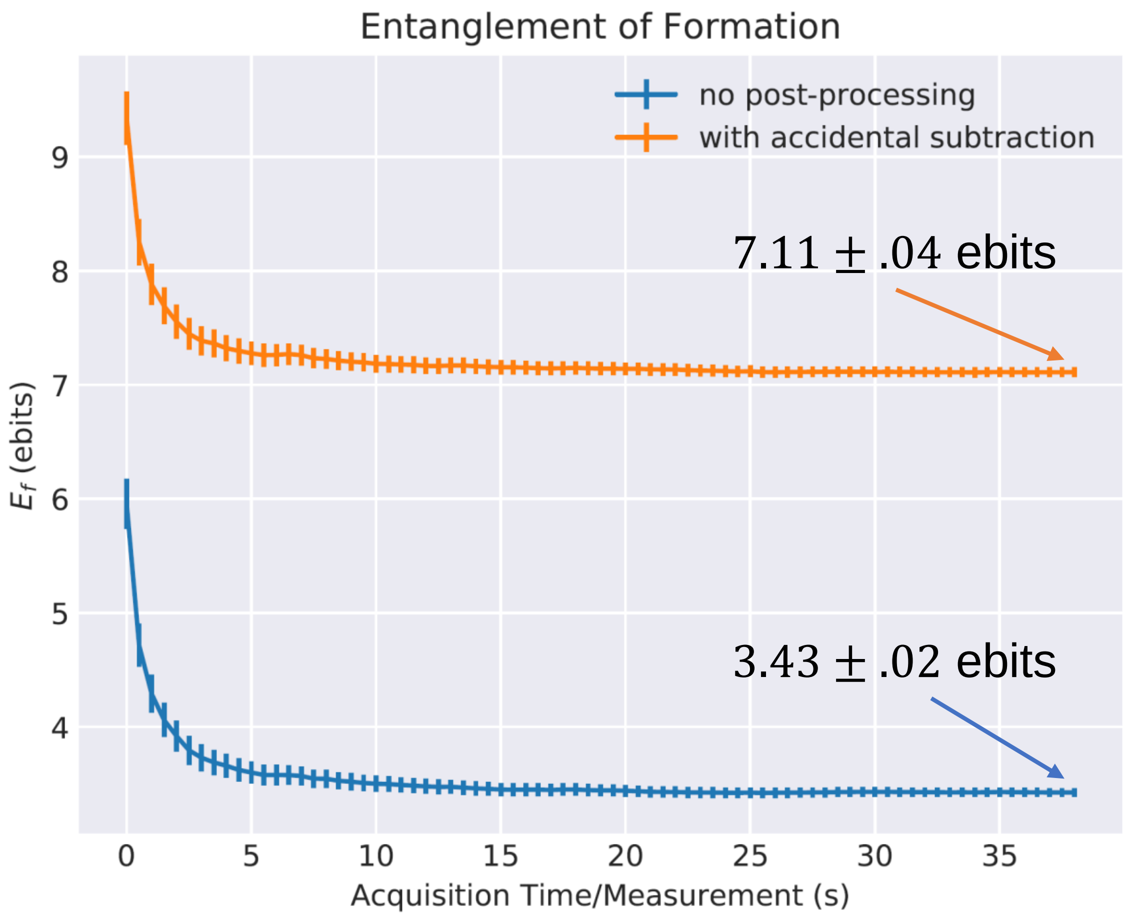
Results: Compressed Distributions



- Separately measure $\tilde{P}(X_A, X_B)$, $\tilde{P}(Y_A, Y_B)$, $\tilde{P}\left(K_A^{(x)}, K_B^{(x)}\right)$, $\tilde{P}(K_A^{(y)}, K_B^{(y)})$

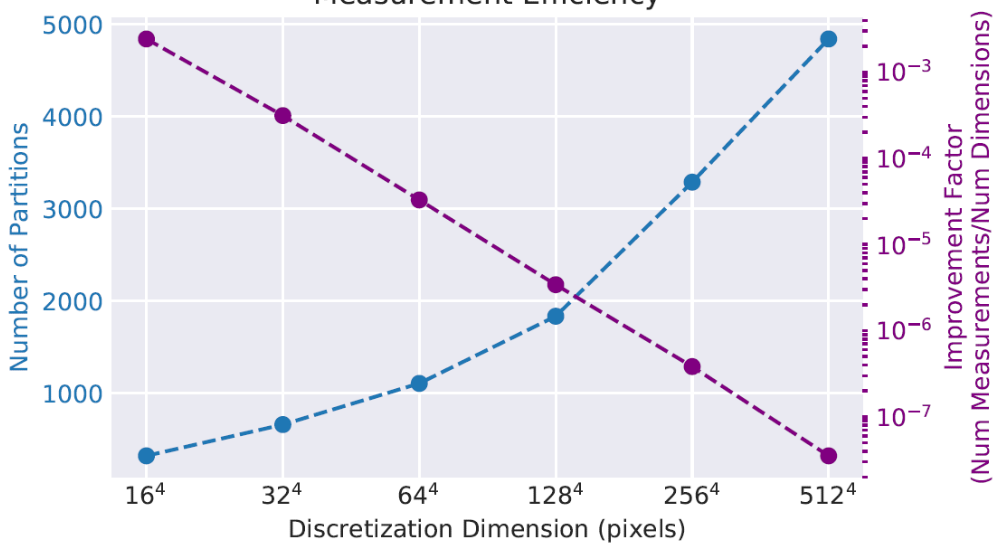
- 512 x 512 pixels-per-photon for each distribution
- 6456 total partitions required (Factor of 20-million fewer than joint-scan)
- Large partitions require ***fewer than 1 coincidence count-per-pixel***

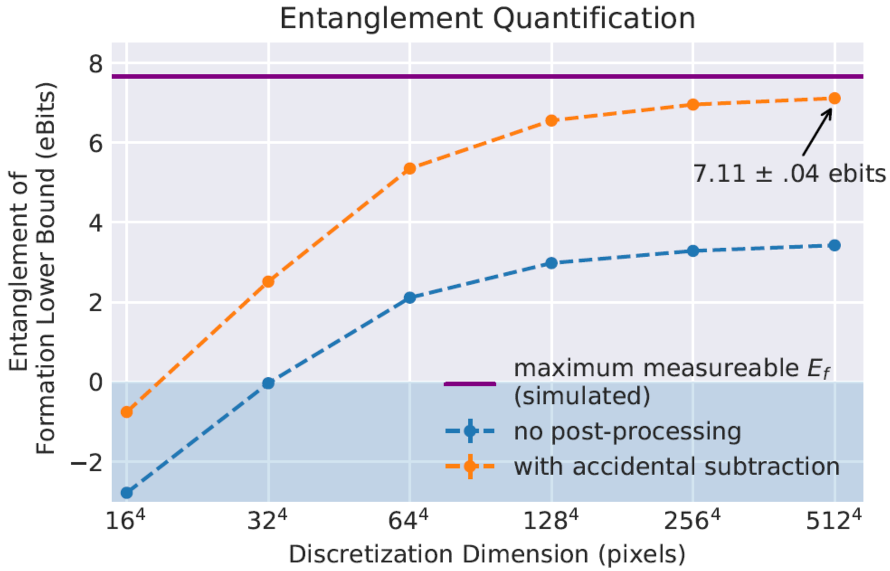
Results: Entanglement vs acquisition time



- Error bars enclose two standard deviations
- Early region is before statistical significance is reached
- 6,456 total measurements

Results: E_f vs maximum resolution





- *Record* 7.1 ebits certified
- 20 million times more efficient than uncompressed version
- 10^18 times more efficient than tomography
- **Progressive** use few measurements to witness entanglement, refine to accurately quantify
- Error bars present but scarcely visible
- No numerical inversion or optimization codes

Full disclosure --- we did make some assumptions in the demo

- Post-selected on coincidence detections
- Did background subtraction (sometimes)
- Have a finite detection window
- Have a finite quantum efficiency of our detectors and other losses

- But this is not the fault of our witness

Other groups are working in this space too

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Quantifying Photonic High-Dimensional Entanglement

Anthony Martin,[1] Thiago Guerreiro,[1,*] Alexey Tiranov,[1] Sébastien Designolle,[1] Florian Fröwis,[1] Nicolas Brunner,[1] Marcus Huber,[1,2] and Nicolas Gisin[1]

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ARTICLES
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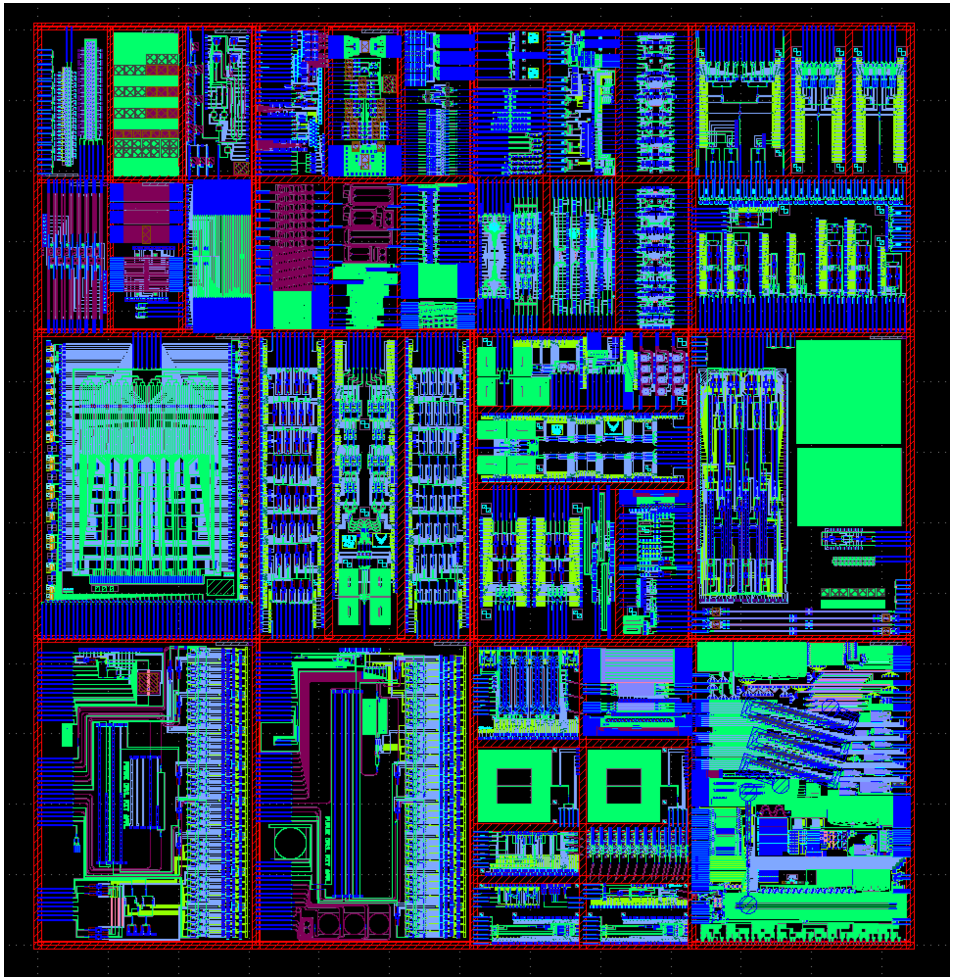
nature physics

Measurements in two bases are sufficient for certifying high-dimensional entanglement

Jessica Bavaresco[1], Natalia Herrera Valencia[1,2], Claude Klöckl[1,3], Matej Pivoluska[1,3,4], Paul Erker[1,5,6], Nicolai Friis[1*], Mehul Malik[1,7*] and Marcus Huber[1*]

Future Work

- Adapt to many-body systems
- Resource-based analysis
  - Instead of how many measurements do we need, how many copies of the quantum system do we consume
- Other ways to use prior knowledge?
- Use multi-channel detection
  - Photon counting cameras
  - Many-channel SNSPD arrays
- Use on interesting platforms (e.g. integrated photonic circuits)



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    - Mike Fanto
    - Chris Tison
    - Paul Alsing
  - Stefan Preble's Group (RIT)

Thank you for listening!

Questions?

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