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Increasing atomic clock precision with and without entanglement

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Outline

- Ramsey Spectroscopy and atomic clocks one atom at a time
- Ramsey spectroscopy with entangled atoms
- Ramsey spectroscopy with *N* atoms using atomic parity measurements
- Photonic interferometry using photon number parity
- Atomic clock based on atomic parity measurements
- How to do the measurements

Noise Reduction in Phase Shift Measurements:

$$\Delta_{\mathrm{SQL}} = 1 \big/ \sqrt{\bar{n}}$$

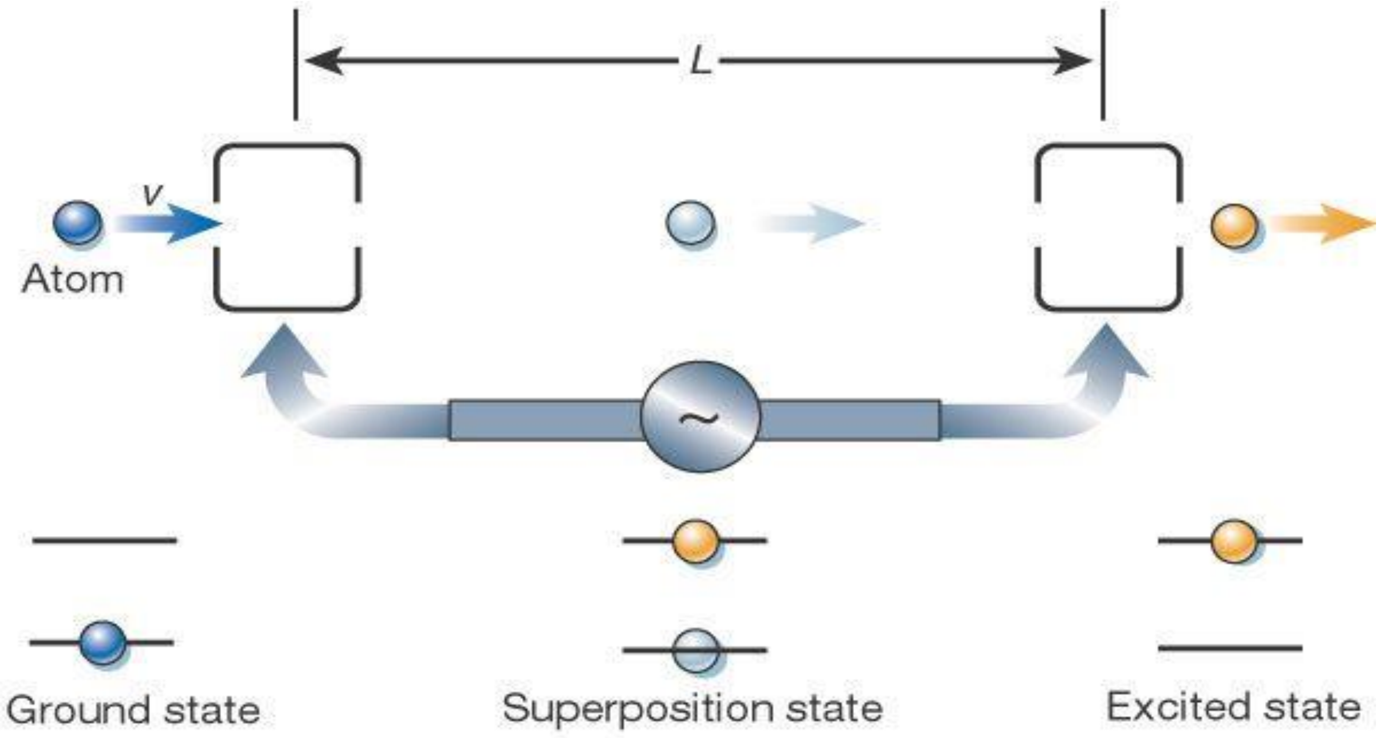
Standard quantum limit
Shot-noise limit

$$\Delta\varphi_{\mathrm{HL}} = 1 \big/ \bar{n}$$

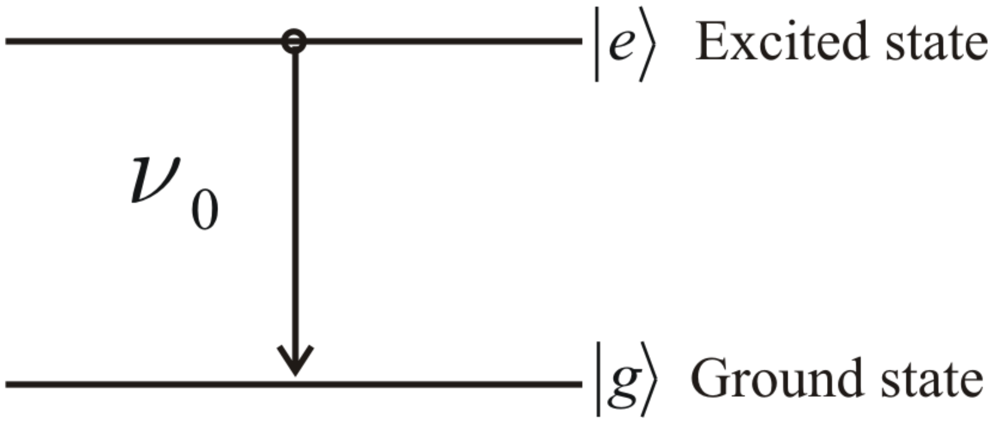
Heisenberg limit

Greatest sensitivity allowed by QM for *linear* phase shifts.

Ramsey method of separated fields



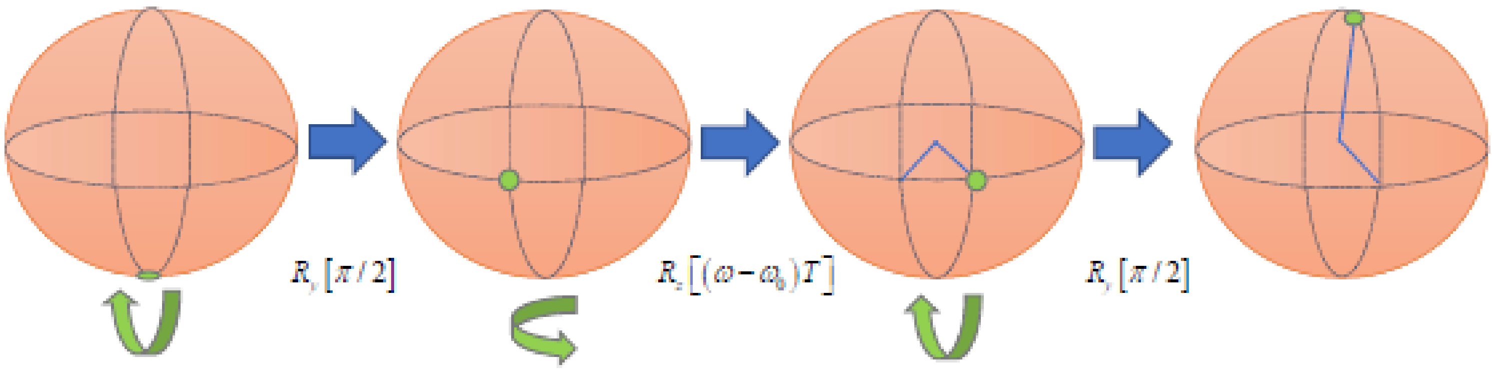
Ramsey spectroscopy with "two-level" atoms or trapped ions:



$$\omega_0 = \frac{E_e - E_g}{\hbar}$$

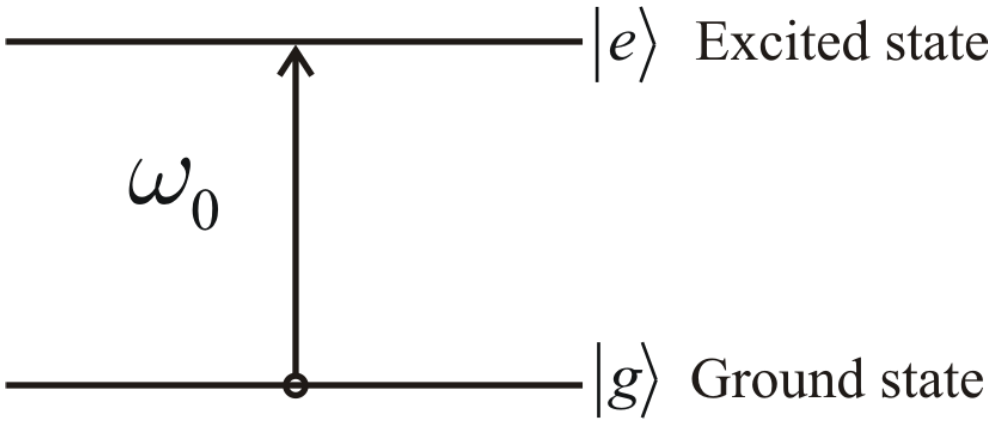
$$\omega_0 = 2\pi\nu_0$$

Bloch sphere picture of Ramsey spectroscopy



Measure probablity of finding atom in the excited state: $P_e = \langle |e\rangle\langle e| \rangle$

Ramsey pulse of frequency ω



Free evolution in frame rotating at ω gives, after time *T*,

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|g\rangle + e^{-i\Delta T}|e\rangle\right)$$

$$\Delta = \omega - \omega_0$$

Second $\frac{\pi}{2}$ Ramsey pulse:

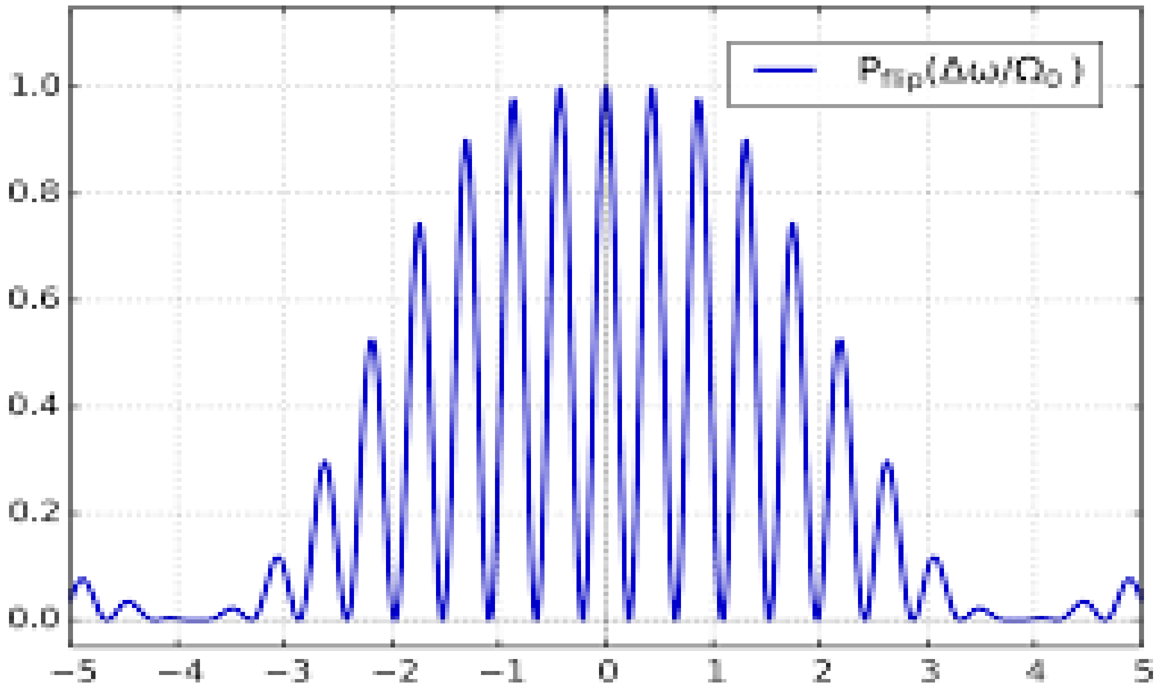
$$|g\rangle \rightarrow \frac{1}{\sqrt{2}}\left(|g\rangle + |e\rangle\right)$$

$$|e\rangle \rightarrow \frac{1}{\sqrt{2}}\left(|e\rangle - |g\rangle\right)$$

$$|\psi\rangle \rightarrow |\psi'\rangle = \frac{1}{2}\left[|g\rangle\left(1-e^{-i\Delta T}\right) + |e\rangle\left(1+e^{-i\Delta T}\right)\right]$$

$$P_e = \left|\langle e|\psi'\rangle\right|^2 = \frac{1}{2}\left\{1+\cos\left[\left(\omega-\omega_0\right)T\right]\right\}$$

Ramsey Fringes



$$\omega - \omega_0$$

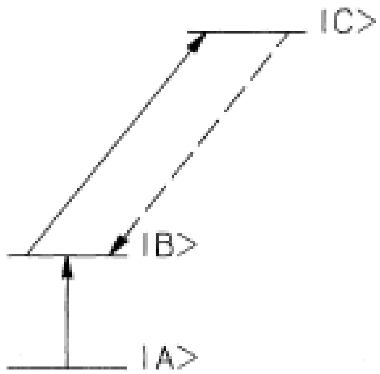
Maximum at $\omega = \omega_0$.

Fringe width (resolution) $\sim \dfrac{1}{T}$

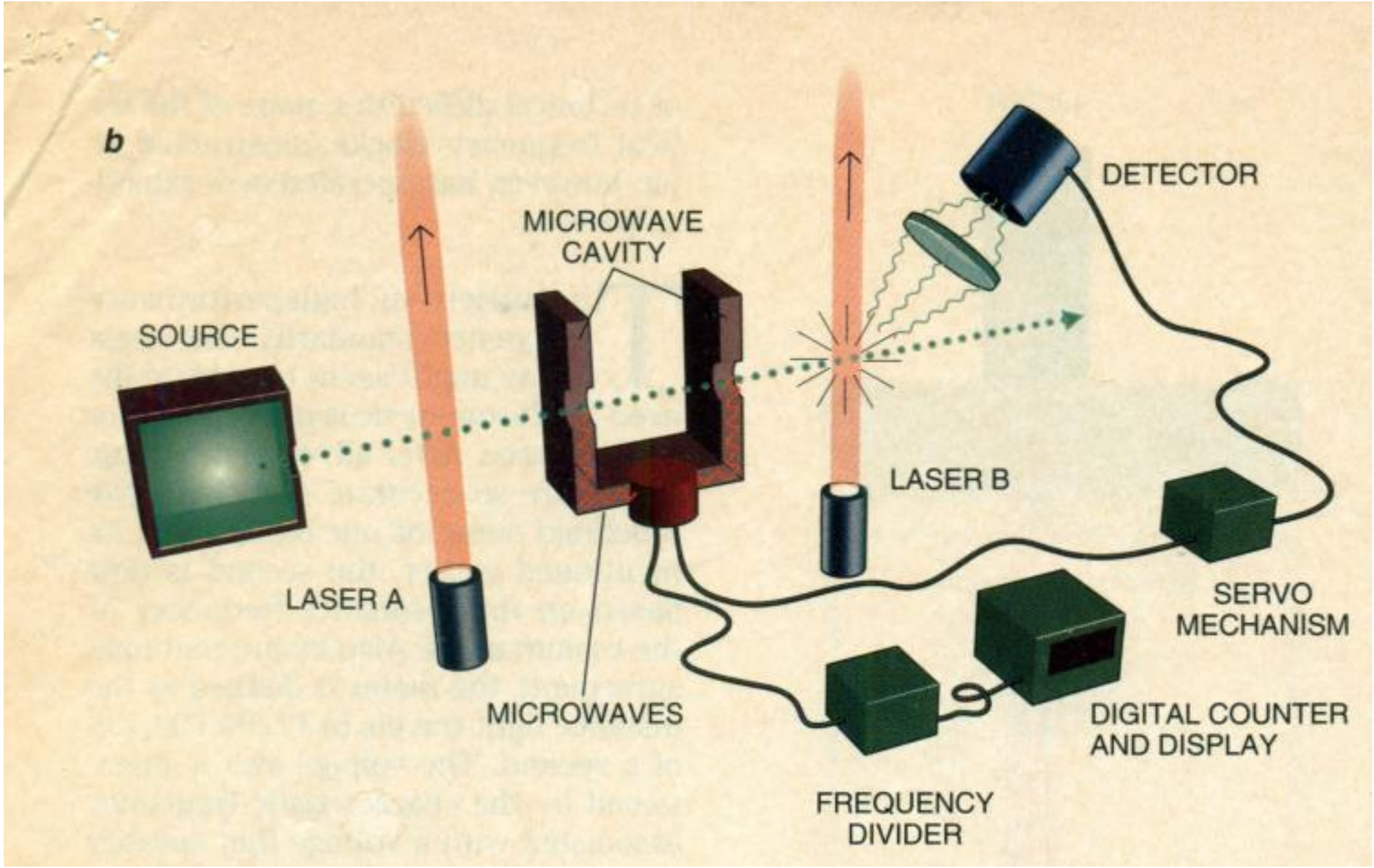
$$\left|\delta\omega_0\right| = \frac{1}{T} \quad \text{one atom}$$

$$\left|\delta\omega_0\right| = \frac{1}{T\sqrt{N}} \quad N \text{ atoms, Standard Quantum Limit (SQL)}$$

Detection is by Dehmelt's electron shelving technique:



Cesium beam atomic clock



Atomic clock definition of the second:

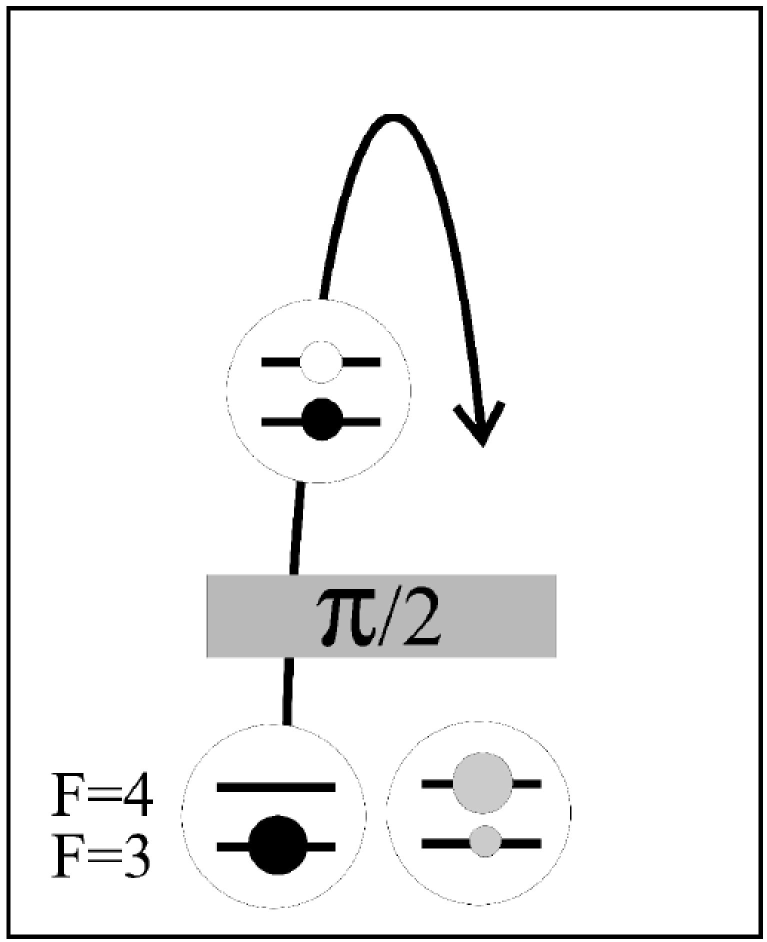
9192 631 770 periods of the radiation for the hyperfine transition:

$$^{133}\mathrm{Cs}:\ F=3, M=0 \rightarrow F=4, M=0.$$

To further improve atomic clocks for greater precision and greater stability, need to make the Ramsey fringes narrower.

One approach: Increase the free evolution time T.

Atomic Fountain clock: increases T



The Entanglement Advantage : Two entangled ions

$$\left|\psi(0)\right\rangle = \frac{1}{\sqrt{2}}\left(\left|g\right\rangle_1 \left|g\right\rangle_2 + \left|e\right\rangle_1 \left|e\right\rangle_2\right) \neq \left|\psi\right\rangle_1 \left|\varphi\right\rangle_2$$

Free evolution for time T :

$$\left|\psi(T)\right\rangle = \frac{1}{\sqrt{2}}\left(\left|g\right\rangle_1 \left|g\right\rangle_2 + e^{-2i\Delta T}\left|e\right\rangle_1 \left|e\right\rangle_2\right)$$

Implement a C-Not gate to disentangle: $\left|e_1\right\rangle\left|e_2\right\rangle \Rightarrow \left|e_1\right\rangle\left|g_2\right\rangle$

$$\left|\psi_{dis}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|g\right\rangle_1 + e^{-2i\Delta T}\left|e\right\rangle_1\right) \otimes \left|g\right\rangle_2$$

$$P_e = \frac{1}{2}\left[1 + \cos(2\Delta T)\right] \Rightarrow \left|\delta\omega_0\right| = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}T} = \frac{1}{2T}$$

For maximally entangled state of *N* ions (J. Steinbach and C.C. Gerry, Phys. Rev. Lett. 81, 5528 (1998)).

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|g\rangle_1 |g\rangle_2 \ldots |g\rangle_N + |e\rangle_1 |e\rangle_2 \ldots |e\rangle_N\right]$$

$$P_e = \frac{1}{2}\left[1 + \cos(N\Delta T)\right]$$

$$|\delta\omega_0| = \frac{1}{NT}$$

Heisenberg-limited

→ **Improvement by a factor** $\frac{1}{\sqrt{N}}$ **over shot-noise limit.**

Alternative Approach (Bollinger et al. PRA 54 (1996)):

Measure parity operator: $\hat{\Pi} = (-1)^{N_e}$

N_e is the number of atoms detected in their excited states

$$\langle \hat{\Pi} \rangle = (-1)^N \cos\left[N(\omega_0 - \omega)T\right]$$
$$= (-1)^N \cos\left[N\Delta T\right]$$

$N =$ total number of ions in the ensemble

Heisenberg limited: $|\delta\omega_0| = \frac{1}{NT}$

Dicke States

$$J_{x,y,z} = \frac{1}{2}\sum_{i=1}^{N} \sigma_{x,y,z}^{i} \qquad \left[J_x, J_y\right] = iJ_z$$

$$\left|J, M\right\rangle, \qquad J = \frac{N}{2}, \quad M = -J, -J+1, \ldots, J$$

$$\left|J, -J\right\rangle = \left|g_1, g_{2,} \ldots, g_N\right\rangle,$$

$$\left|J, -J+1\right\rangle = \frac{1}{\sqrt{N}}\left[\left|e_1, g_{2,} \ldots, g_N\right\rangle + \left|g_1, e_{2,} \ldots, g_N\right\rangle + \ldots + \left|g_1, g_{2,} \ldots, e_N\right\rangle\right],$$

...

$$\left|J, J\right\rangle = \left|e_1, e_{2,} \ldots, e_N\right\rangle,$$

Atomic Coherent States

$$R(\theta,\phi)=\exp\left(zJ_+ - z^* J_-\right)$$

$$\left|\zeta,J\right\rangle = R(\theta,\phi)\left|J,-J\right\rangle = \left(1+\left|\zeta\right|^2\right)^{-J}\sum_{M=-J}^{J}\binom{2J}{J+M}^{1/2}\zeta^{J+M}\left|J,M\right\rangle,$$

$$z=e^{-i\phi}\,\theta/2,\zeta=e^{-i\phi}\tan(\theta/2).$$

$$\left|\zeta,J\right\rangle \equiv \left|\theta,\phi\right\rangle_J = \otimes_{i=1}^{N}\left[e^{-i\phi/2}\sin(\theta/2)\left|e\right\rangle_i + e^{i\phi/2}\cos(\theta/2)\left|g\right\rangle_i\right]$$

First $\frac{\pi}{2}$ pulse:

$$\left|-1,J\right\rangle = \exp\left(-i\tfrac{\pi}{2}J_y\right)\left|J,-J\right\rangle = 2^{-J}\sum_{M=-J}^{J}\binom{2J}{J+M}^{1/2}(-1)^{J+M}\left|J,M\right\rangle,$$

$$\left|-1,J\right\rangle = \otimes_{i=1}^{N}\left[-i\left(\left|g\right\rangle_i - \left|e\right\rangle_i\right)/\sqrt{2}\right].$$

Initial State: $\left|J,-J\right\rangle$, all atoms in their ground states.

$$\hat{U}(\varphi)=\exp\left(-i\frac{\pi}{2}\hat{J}_y\right)\exp\left(-i\varphi\hat{J}_z\right)\exp\left(-i\frac{\pi}{2}\hat{J}_y\right).$$

$$2\left\langle\hat{J}_z(\varphi)\right\rangle=2\left\langle J,-J\right|\hat{U}^{\dagger}(\varphi)\hat{J}_z\hat{U}(\varphi)\left|J,-J\right\rangle=2J\cos\varphi$$

$$=N\cos\left[\left(\omega-\omega_0\right)T\right],$$

Suppose we prepare the state

$$\left|\Psi\right\rangle = \frac{1}{\sqrt{2}}\left[\left|J,J\right\rangle + \left|J,-J\right\rangle\right]$$

Creation of this state replaces the first Ramsey pulse.

Final state is then

$$\left|\Psi_f\right\rangle = \exp\left(-i\frac{\pi}{2}J_y\right)\exp\left(-i\varphi J_z\right)\left|\Psi\right\rangle$$

$$e^{-i\varphi J_z}\left|\Psi\right\rangle = \frac{1}{\sqrt{2}}\left[e^{-iN\varphi/2}\left|J,J\right\rangle + e^{iN\varphi/2}\left|J,-J\right\rangle\right]$$

$$\left\langle\Psi_f\right|J_z\left|\Psi_f\right\rangle = 0$$

Collective atomic, or SU(2), parity operator

$$\Pi_J = \exp\left[i\pi\left(J - J_z\right)\right]$$

No classical analog!

$$\left\langle \Psi_f \middle| \Pi_J \middle| \Psi_f \right\rangle = \left(-1\right)^{2J} \cos\left[2J\left(\omega - \omega_0\right)T\right]$$

$$= \left(-1\right)^{N} \cos\left[N\left(\omega - \omega_0\right)T\right]$$

Definition of photon number parity

Photon number operator:

$$\hat{n} = \hat{a}^{\dagger}\hat{a}, \quad \left[\hat{a}, \hat{a}^{\dagger}\right] = 1$$

$$\hat{n}\left|n\right\rangle = n\left|n\right\rangle, \quad n = 0, 1, 2..., \infty$$

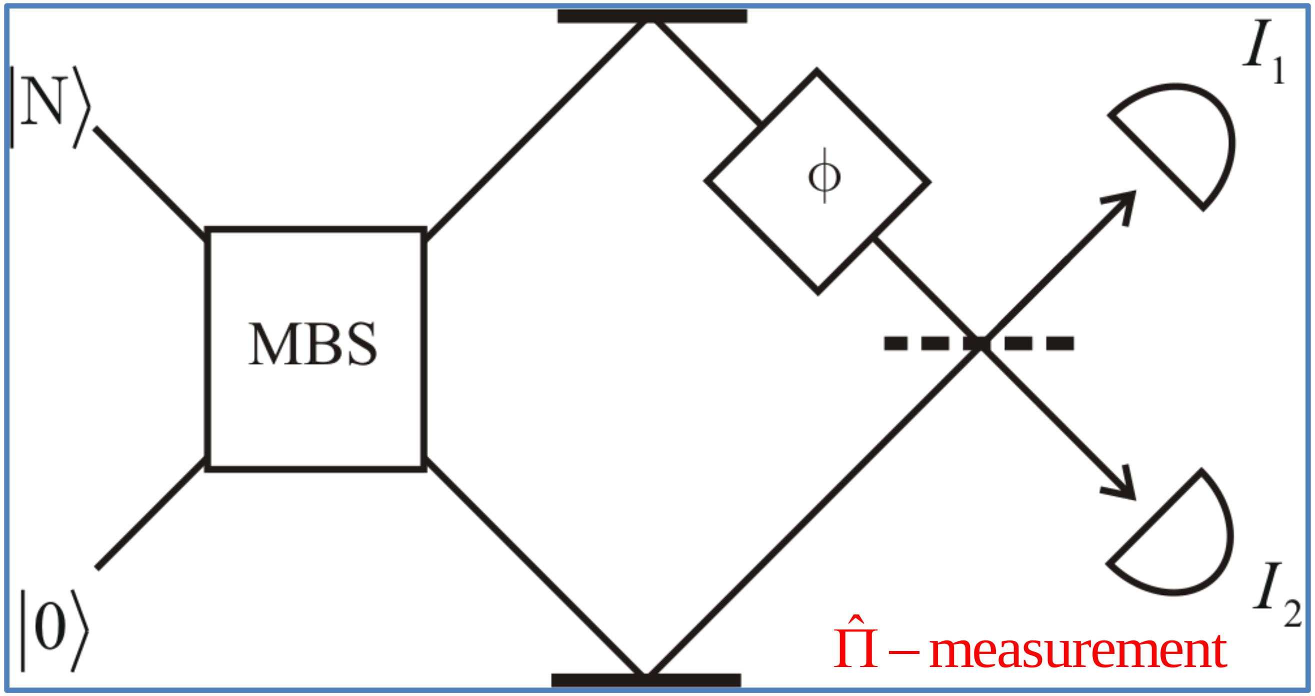
Parity Operator:

$$\hat{\Pi} = \left(-1\right)^{\hat{n}} = \exp\left(i\pi\hat{n}\right)$$

No classical analog!

$$\hat{\Pi}\left|n\right\rangle = \left(-1\right)^{n}\left|n\right\rangle$$

Interferometry with a N00N state?

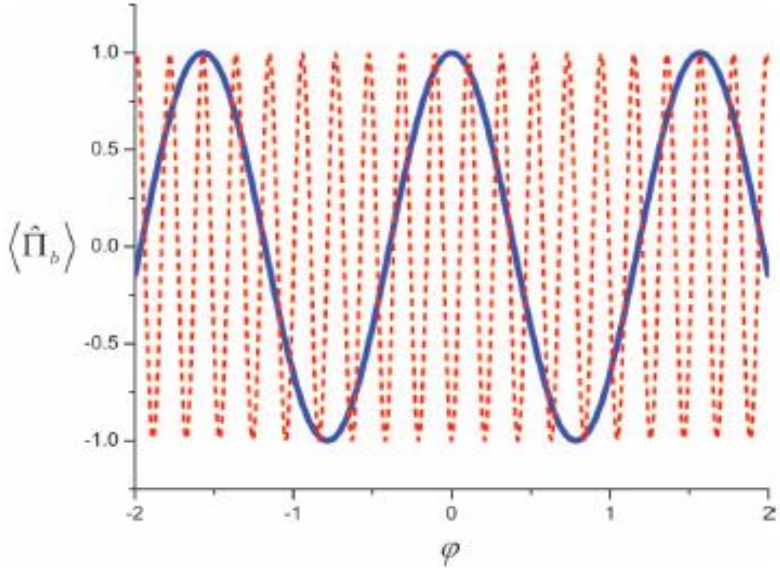


$$|\Psi_N\rangle = \frac{1}{\sqrt{2}}\left[|N\rangle|0\rangle + |0\rangle|N\rangle\right]$$

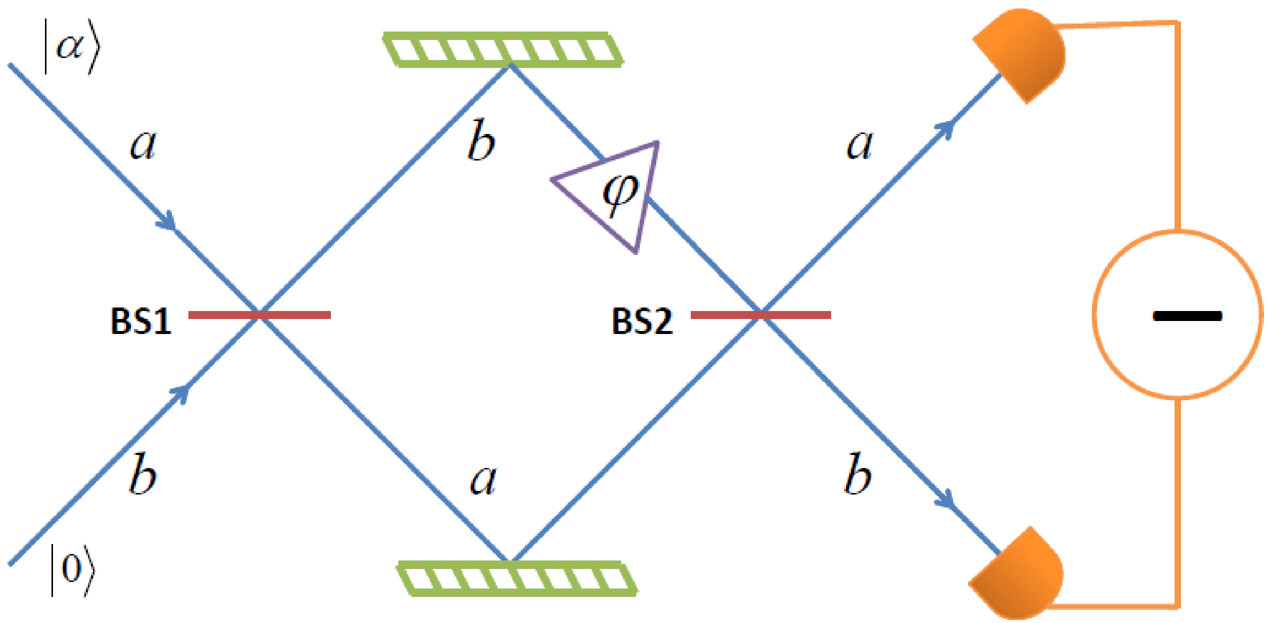
$$\left\langle \hat{\Pi} \right\rangle = \begin{cases} (-1)^{N/2} \cos\left(N\varphi + \Phi_N\right), & N \text{ even}, \\ (-1)^{(N+1)/2} \sin\left(N\varphi + \Phi_N\right), & N \text{ odd}. \end{cases}$$

$$\Delta\varphi = \frac{\Delta\Pi}{\left|\partial\langle\Pi\rangle/\partial\varphi\right|} = \frac{1}{N}$$ Heisenberg-limited

Super-resolved interference fringes



Interferometry with coherent light



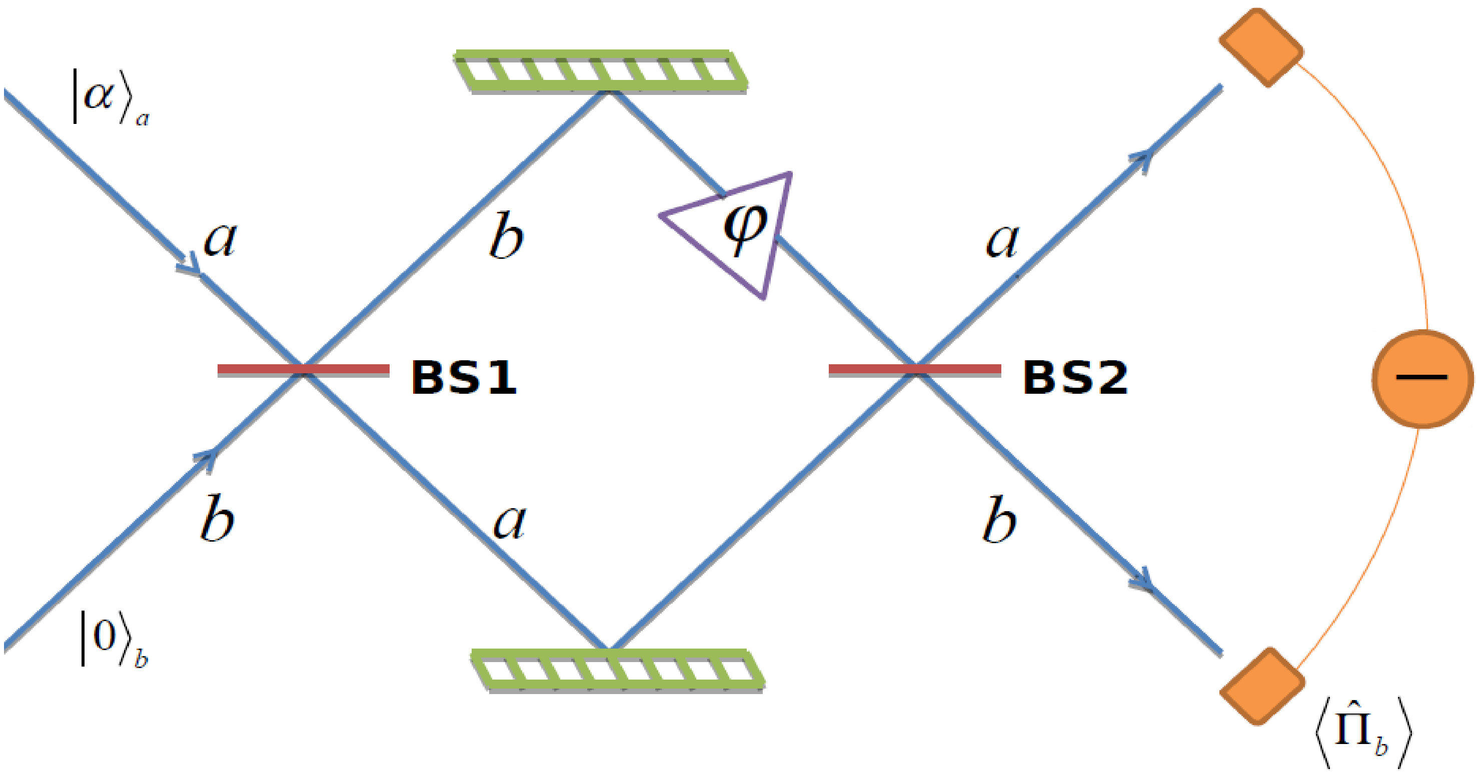
$$S = \langle \hat{a}^\dagger \hat{a} - \hat{b}^\dagger \hat{b} \rangle = -\bar{n}\cos\varphi$$

$$\Delta\varphi = \frac{\Delta S}{|\partial S/\partial\varphi|} = \frac{1}{\sqrt{\bar{n}}\,|\sin\varphi|}$$

$$\Delta\varphi_{\mathrm{SQL}} = \frac{1}{\sqrt{\bar{n}}}$$ Standard quantum limit

Coherent light interferometry and parity measurements

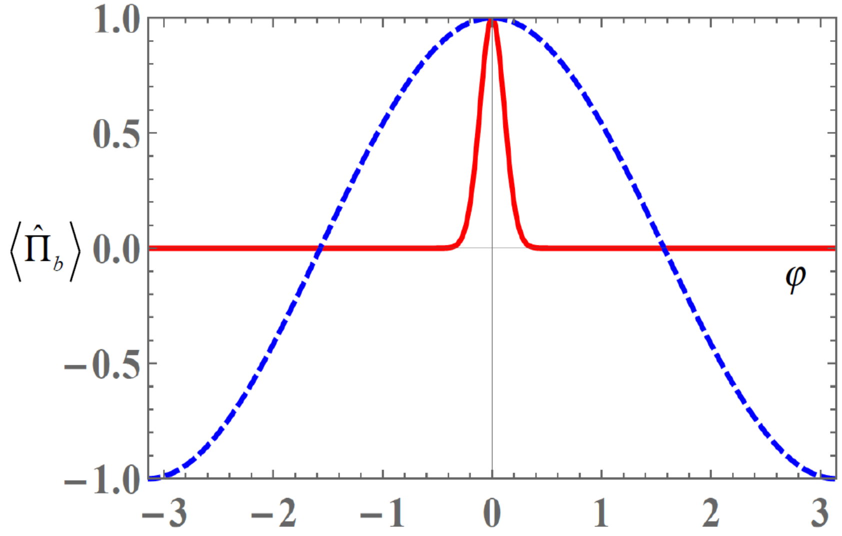
Dowling and collaborators



Photon number parity approach to interferometry with coherent light

$$\left\langle \hat{\Pi}_b(\varphi) \right\rangle_0 = \exp\left[-\bar{n}\left(1-\cos\varphi\right)\right]$$

Super-resolved:



$$\Delta\varphi_{\min} = \frac{1}{\sqrt{\bar{n}}\left|\cos\varphi\right|}$$

Experiment: L. Cohen *et al*. Opt. Exp. **22**, 011945 (2014)

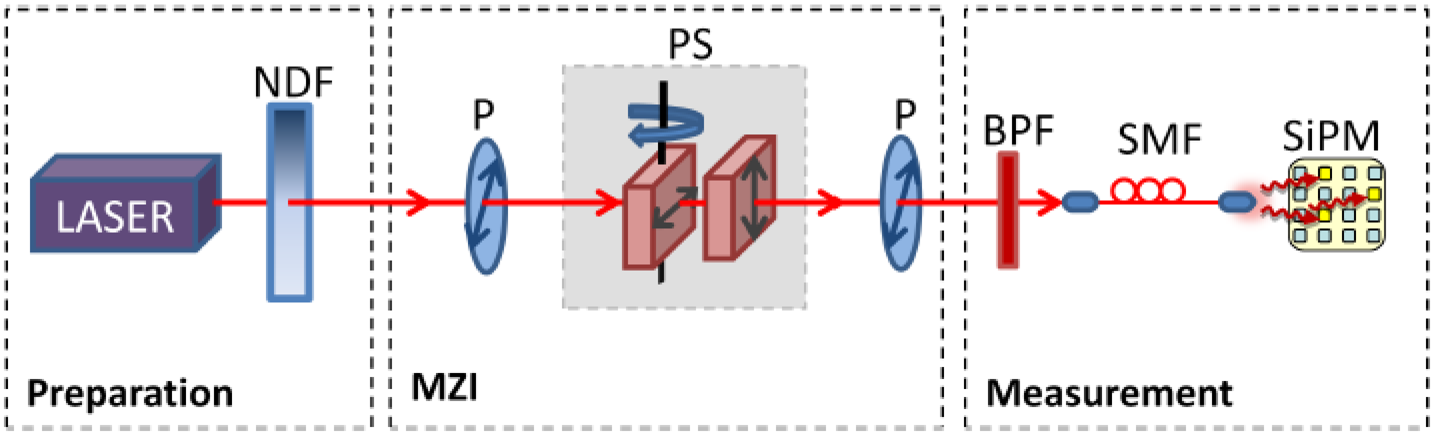


Fig. 1. The experimental setup. The coherent states are produced by a Ti:Sapphire laser and their average photon number is controlled by a calibrated variable neutral density filter (NDF). The Mach-Zehnder interferometer (MZI) is composed from two polarizers (P) at 45° and a phase shifter (PS). One of the output modes from the MZI is filtered spectrally by $3\,nm$ band pass filter (BPF) and spatially by a single mode fiber (SMF). This mode is detected by the silicon photomultiplier (SiPM, *Hamamatsu Photonics*, S10362-11-100U) detector.

From L. Cohen *et al.* Opt. Exp. **22**, 011945 (2014)

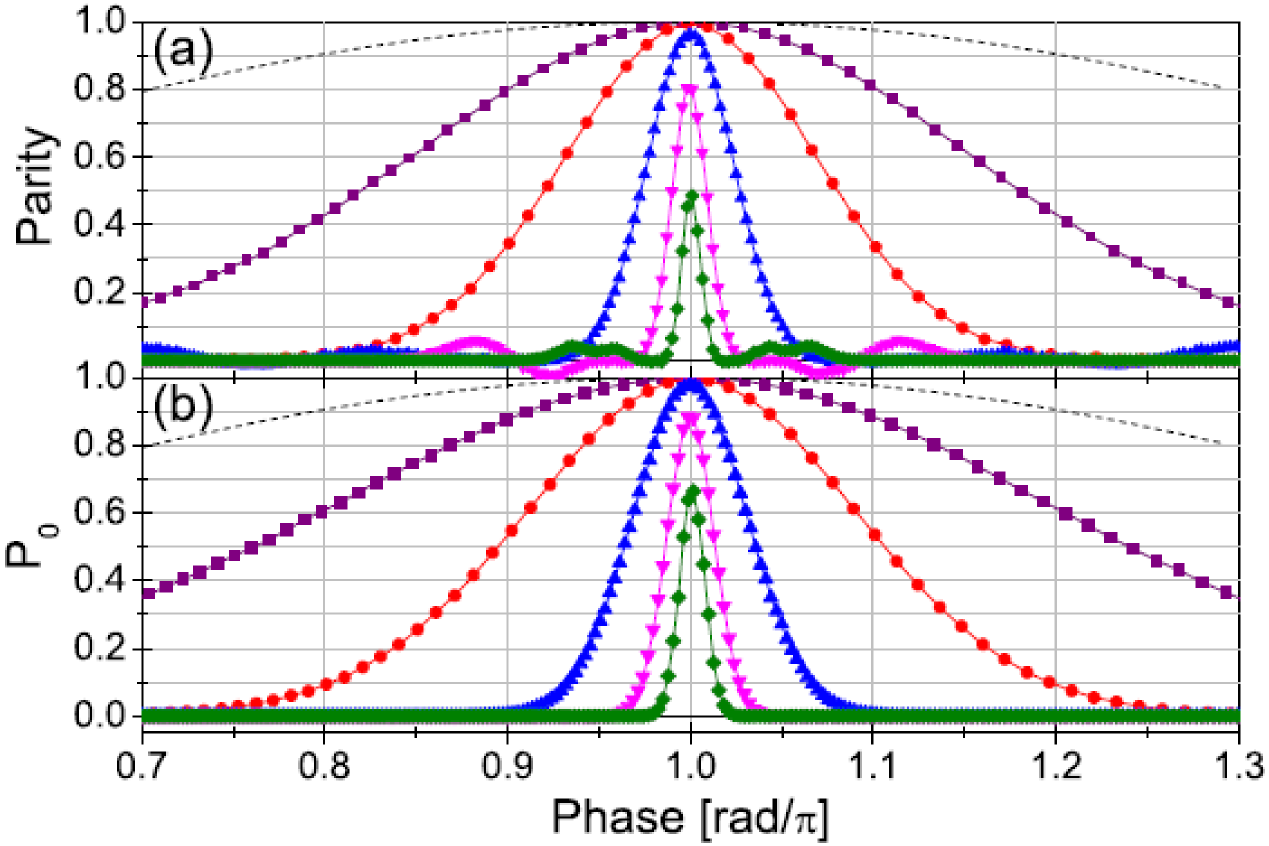
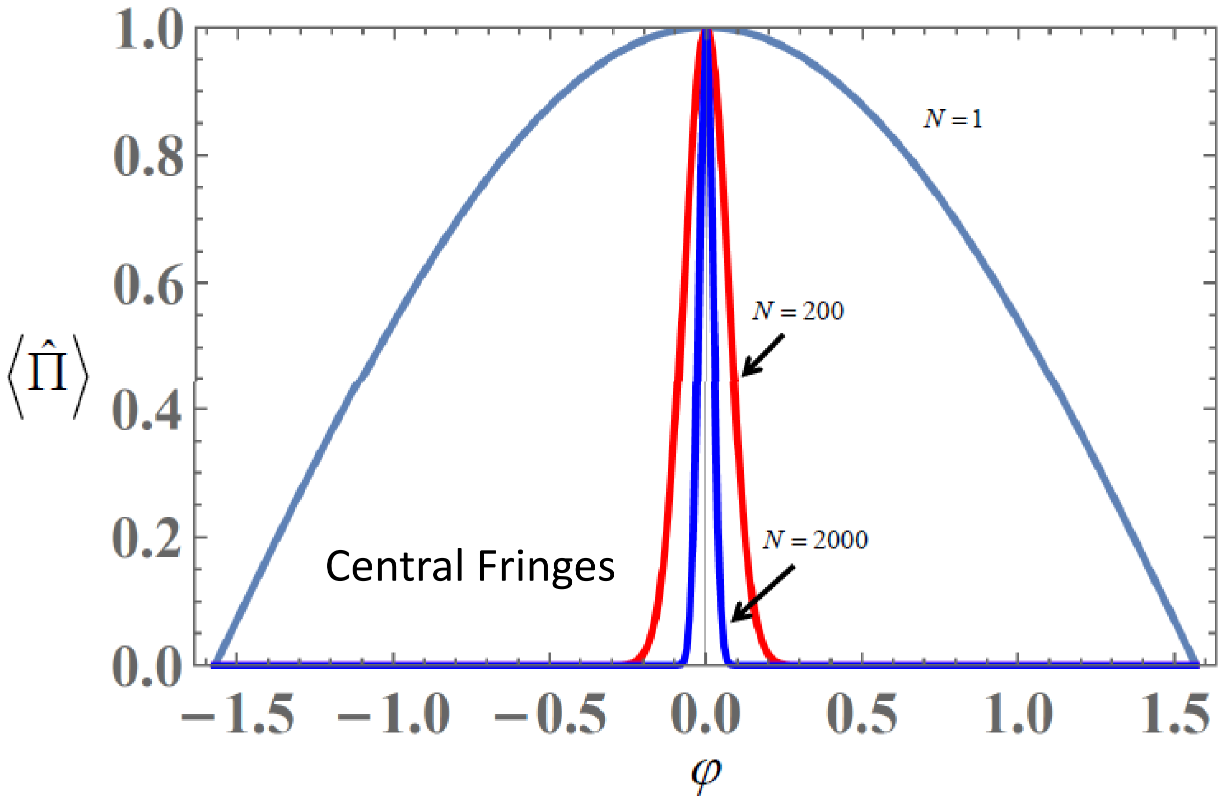


Fig. 2. (a) Parity and (b) P_0 dependence on the phase. The presented measurements are for average photon numbers of 4.6 ± 0.2 (purple squares), 25 ± 1 (red circles), 200 ± 8 (blue triangles), $1,190 \pm 50$ (pink inverted triangles),and $4,150 \pm 150$ (green rhombuses). The dashed black lines represent the classic interference curves, and are presented for comparison reasons. Errors are not shown, as they are smaller than the symbols.

For the initial state $|g\rangle_1 |g\rangle_2 \cdots |g\rangle_N$

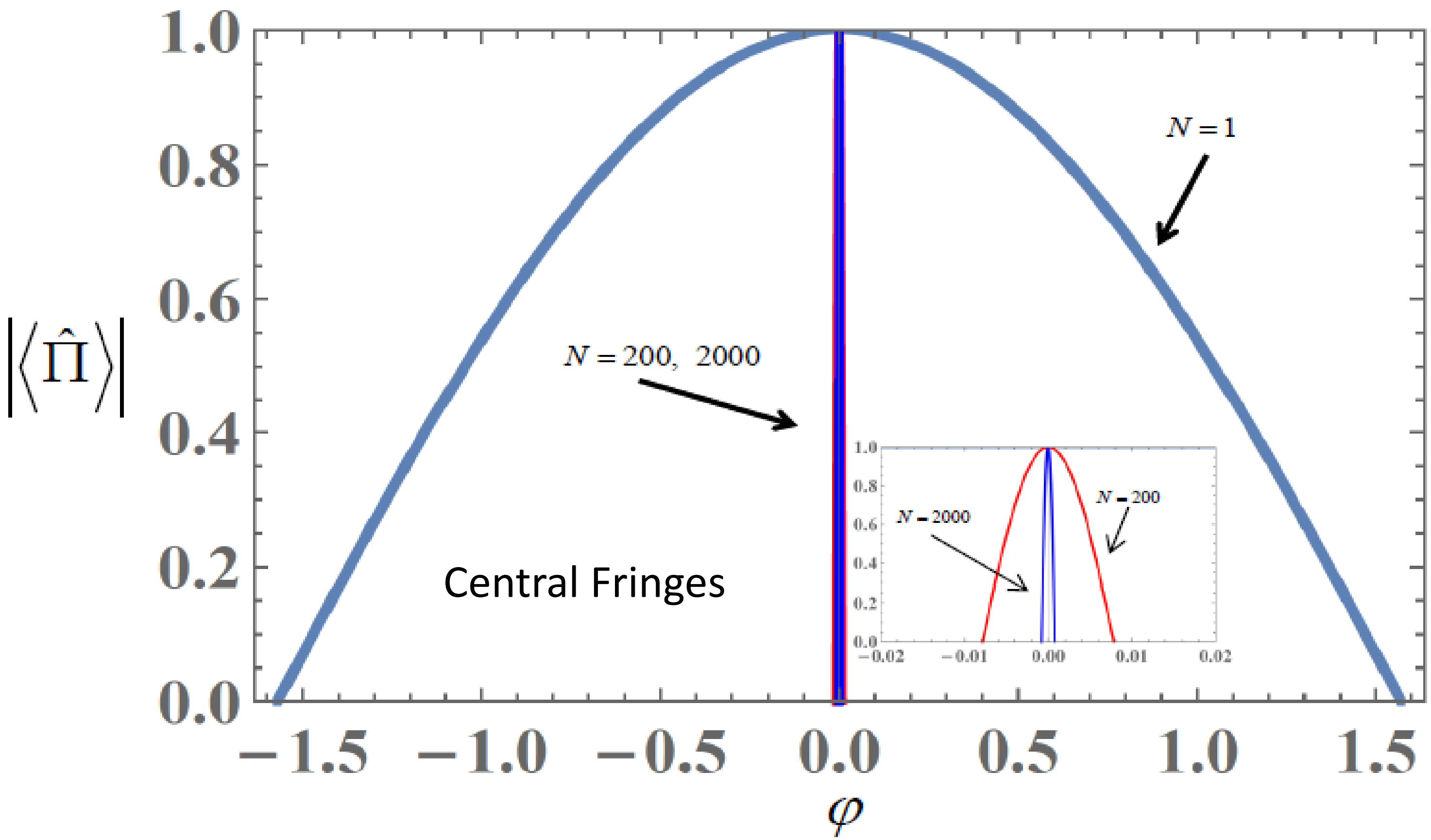


$$\langle \hat{\Pi} \rangle = \cos^N \left[(\omega - \omega_0) T \right]$$

There is no entanglement involved!

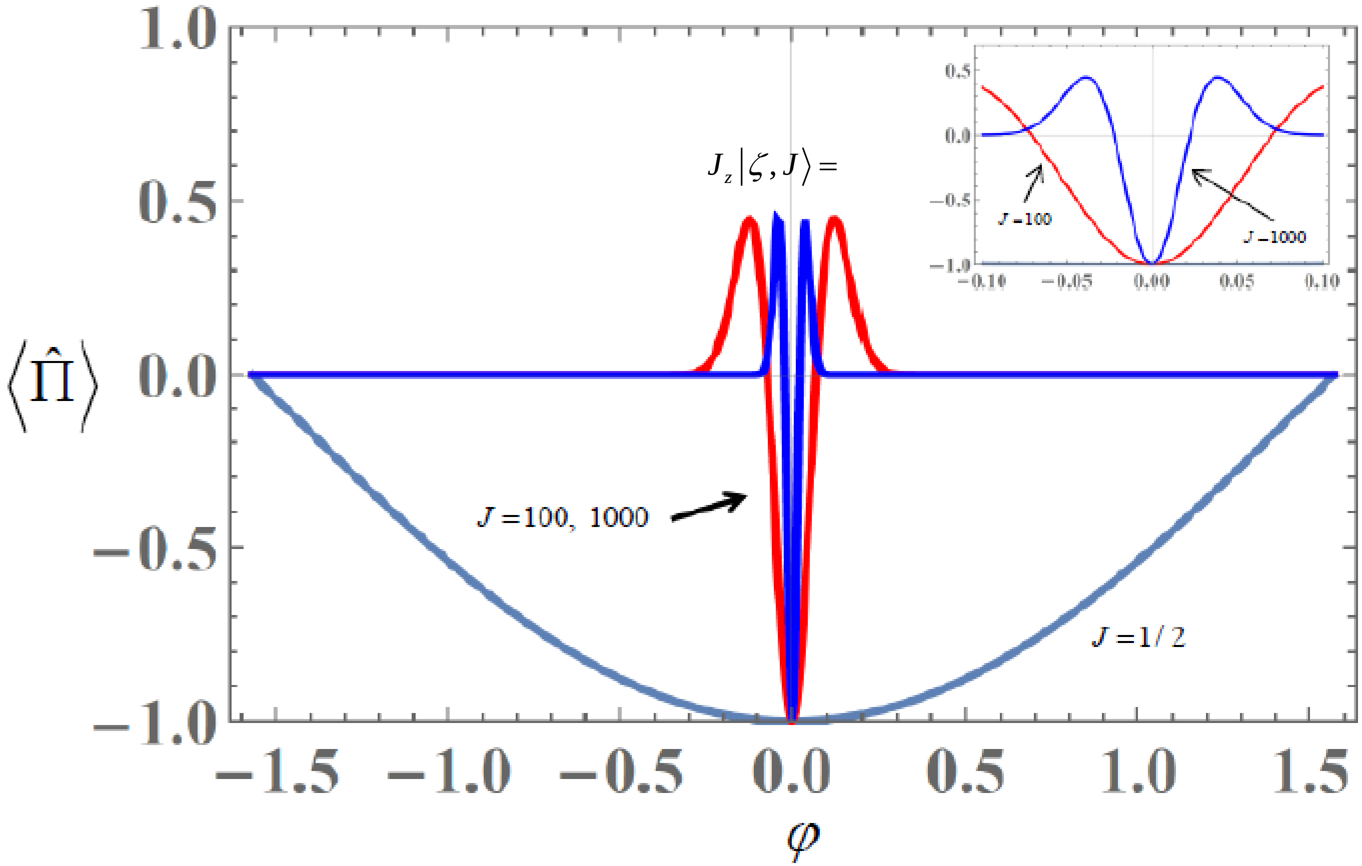
For the maximally entangled state:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|g\rangle_1 |g\rangle_2 \ldots |g\rangle_N + |e\rangle_1 |e\rangle_2 \ldots |e\rangle_N\right] = \frac{1}{\sqrt{2}}\left[|J-J\rangle + |J,J\rangle\right]$$



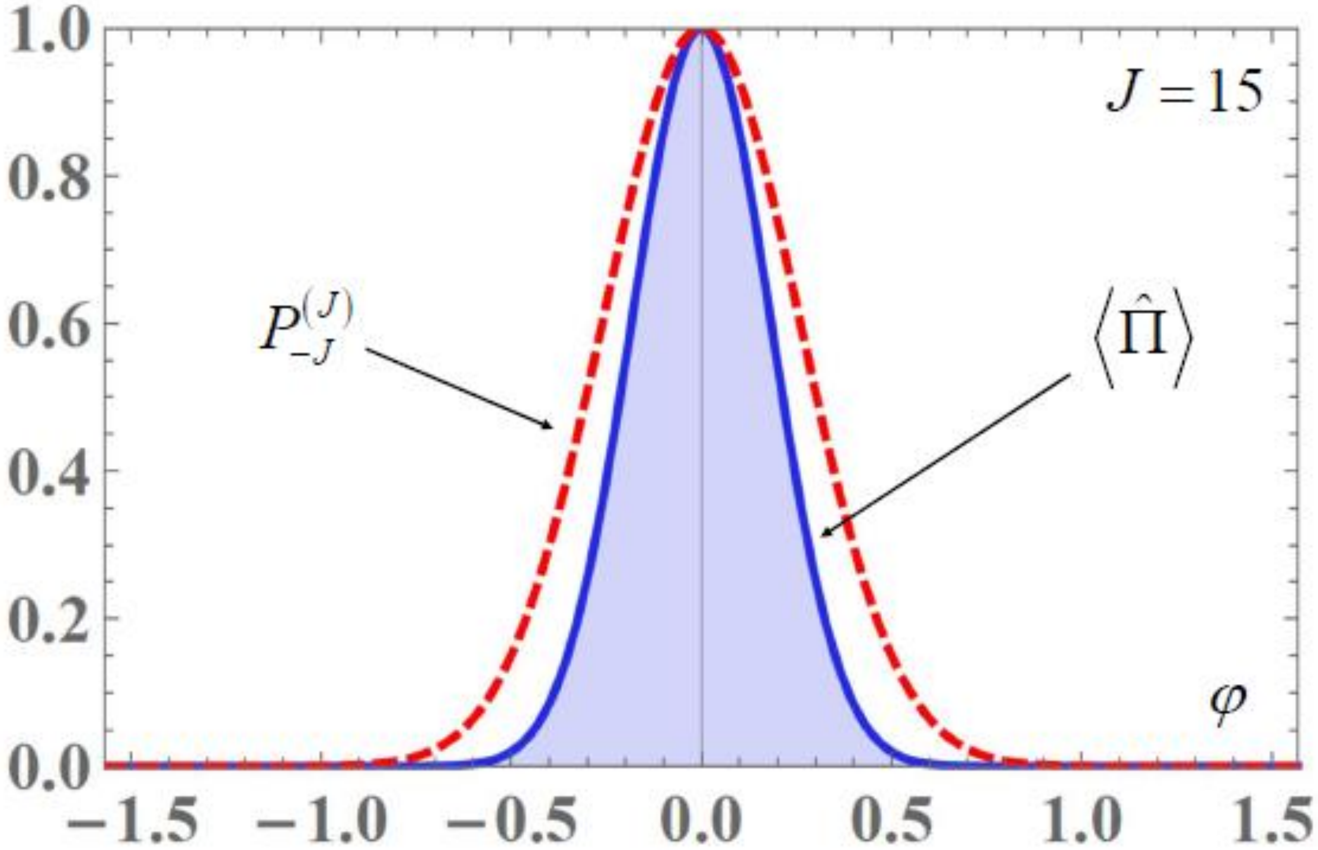
$$J_z\left|-1,J\right\rangle \Rightarrow \left|\psi\right\rangle = \mathcal{N}\sum_{M=-J}^{J}\binom{2J}{J+M}^{1/2} M\,(-1)^{J+M}\left|J,M\right\rangle$$

Entangled state.



Alternative: Prob. of detecting $|J,-J\rangle$

$$\Pi_M^J = |J,M\rangle\langle J,M|, \qquad P_M^{(J)} = \langle \text{out}|\Pi_M^J|\text{out}\rangle$$



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Quantum nondemolition measurement of parity and generation of parity eigenstates in optical fields

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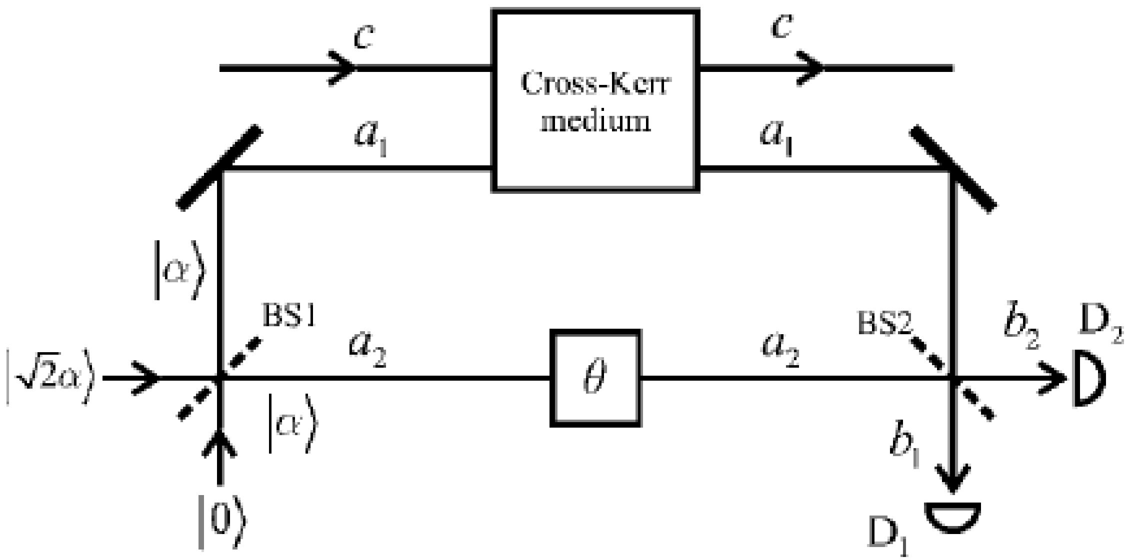

$$\chi \hat{n}_1 \hat{n}_c \rightarrow \lambda \hat{n}_1 \hat{J}_z$$