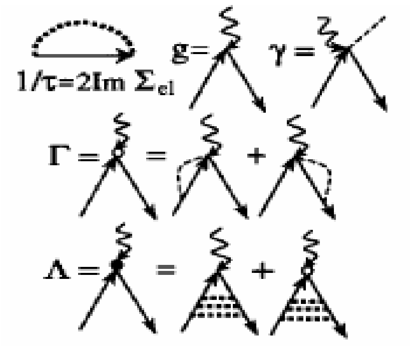


Quantum Noise Limit in Ultrasensitive Nanodetectors: Counting of Phonons

Andrei Sergeev
SUNY at Buffalo

Every photon should be counted –

as many as possible phonons should be uncounted!

How to suppress (manage) e-ph thermal conductance?

- **Low temperatures,**
- **Small device volume,**
- **Suppression of e-ph coupling constant,**
- **Phonon engineering.**

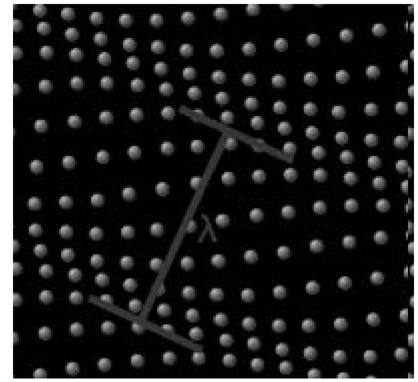
Questions addressed:
What is the origin of the e-ph interaction in metallic (superconducting) nanostructures?
Can we control e-ph coupling?

e-ph thermal conductance in the limit of weak coupling

“The noise is the signal”
Rolf Landauer

Questions addressed:
Can we observe single e-ph scattering events,
i.e. the “quantum limit” for the e-ph relaxation?
Electron relaxation in the quantum limit?
Noise in the quantum limit?

E-Ph Interaction in Pure Bulk Materials
Deformation Potential



- Phonons are elementary excitations, which describe vibrations of the lattice;
- Vibrations result in the lattice deformation;
- Deformation modifies the charge distribution, which leads to coupling between electrons and phonons.

Deformation potential in metals and semiconductors has different origin.
Metals: electron gas compressibility (change in the local concentration leads to a shift of the local Fermi energy);
Semiconductors: deformation shifts the conduction band edge.

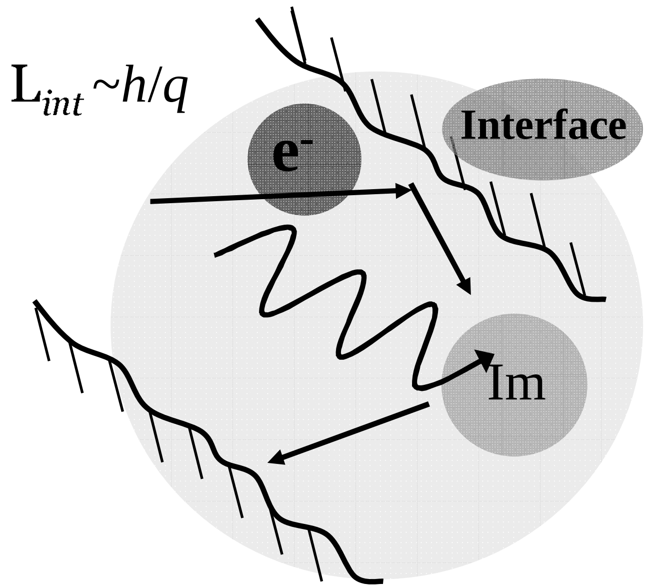
In the isotropic model, only longitudinal vibrations (phonons) interact with electrons, because transverse vibrations do not change the volume of a sell (local concentration).

Metals: $D = g\,(1 - x^2/3)$,
x is cos between **p** and **q**;
Semiconductors: $D = D_0$

p is the electron momentum
q is the phonon wave vector

Electron scattering from vibrating impurities and boundaries: Interference of scattering mechanisms

Electron "simultaneously" scatters from a phonon, interfaces, and a dopant.



The interaction region is ~ h/q,
q is the transferred momentum,
In bulk conductors $q=T/u$, and
L_{int} is ~ phonon wavelength

Ziman: "it became impossible to establish proper interference condition for the quasi-momenta in a scattering process, and the formulation in terms of separate scattering events then broke down."

Interfaces, boundaries, defects, dopants, impurities = Disorder
(Vibrating Disorder!)
ℓ is the *elastic* mean free path

$q\ell < 1$ **is the quasi-ballistic limit,**

$q\ell > 1$ **is the diffusive limit**

$T_{cr}(\ell = 10\text{ nm}) \sim 1\text{K}$

- ***In the quasi-ballistic limit: deformation interaction + scattering from vibrating impurities and boundaries;***
- ***In the diffusive limit: interference strongly affects the electron relaxation and transport.***

Sir Brian Pippard
1920-2008

Pippard Ineffectiveness Condition

Method: Special frame moving with a phonon (Tsuneto transformation)

Ziman: "Pippard's method is quite different from those with which we are familiar (the transport equation). Pippard's argument may be called kinetic in principle."

In the diffusive limit the relaxation rate is a factor of $q\ell = T\ell/u$ slow than that in the pure limit.

Longitudinal phonons | **Transverse phonons**

- **Quasi-ballistic limit**, $q_T\ell = T\ell/u >> 1$ ($q_T = T/u$)

$$\frac{1}{\tau_{e-l.ph}} = \frac{7\pi\varsigma(3)}{2}\frac{\beta_l T^3}{(p_F u_l)^2}$$

$$\frac{1}{\tau_{e-t.ph}} = \frac{3\pi^2\beta_t T^2}{(p_F u_t)(p_F\ell)} \sim \frac{1}{\tau_{e-l.ph}(q_T\ell)}$$

- **Diffusive limit**, $q_T\ell < 1$,

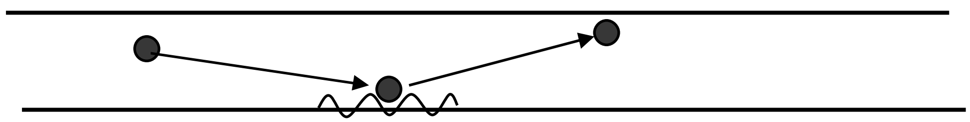
$$\frac{1}{\tau_{e-l.ph}} = \frac{\pi^4\beta_l}{5}\frac{p_F\ell T^4}{(p_F u_l)^3}$$

$$\frac{1}{\tau_{e-t.ph}} = \frac{3\pi^4\beta_t}{10}\frac{p_F\ell T^4}{(p_F u_t)^3}$$

p_F is the Fermi momentum, u_l and u_t are longitudinal and transverse sound velocities, coupling constants are related as $\beta_l/\beta_t = (u_t/u_l)^2$

Details can be found in Int.J.Mod.Phys. 10, 635 (1996).

Transverse Phonons in Quasi – Ballistic Limit



Pippard model:
Boundaries vibrate as host atoms

- In the quasi-ballistic limit, the longitudinal phonons dominate over transverse phonons in the relaxation, when

$$T > T_{tr} = \frac{6\pi}{7\varsigma(3)}\left(\frac{u_l}{u_t}\right)^3\left(\frac{\hbar u_l}{k_B \ell}\right)$$

- In the most pure metallic films the electron mean free path ℓ is determined by the electron-boundary scattering, so ℓ is of the order of the film thickness, *d*.

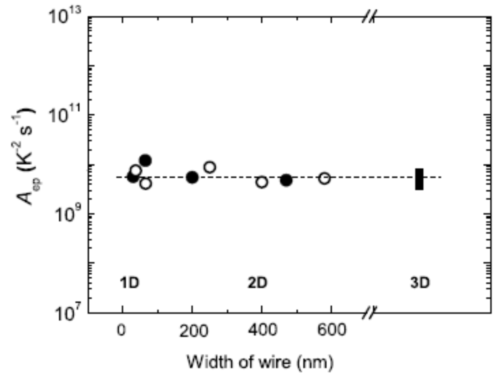
- Taking typical data, $u_l / u_t = 2$
  (for example in Al, $u_l = 6.3*10^5$ cm/sec , $u_t = 3.1*10^5$ cm/sec) and $\ell \sim d = 30$ nm,
  we get: **$T_{tr} \sim 40$ K**!

- In pure metallic films the electron interaction with transverse phonons is realized due to vibrating boundaries.
  For *d* ~ 30 nm, below T < 40 K, e-ph interaction is solely determined by vibrating boundaries!

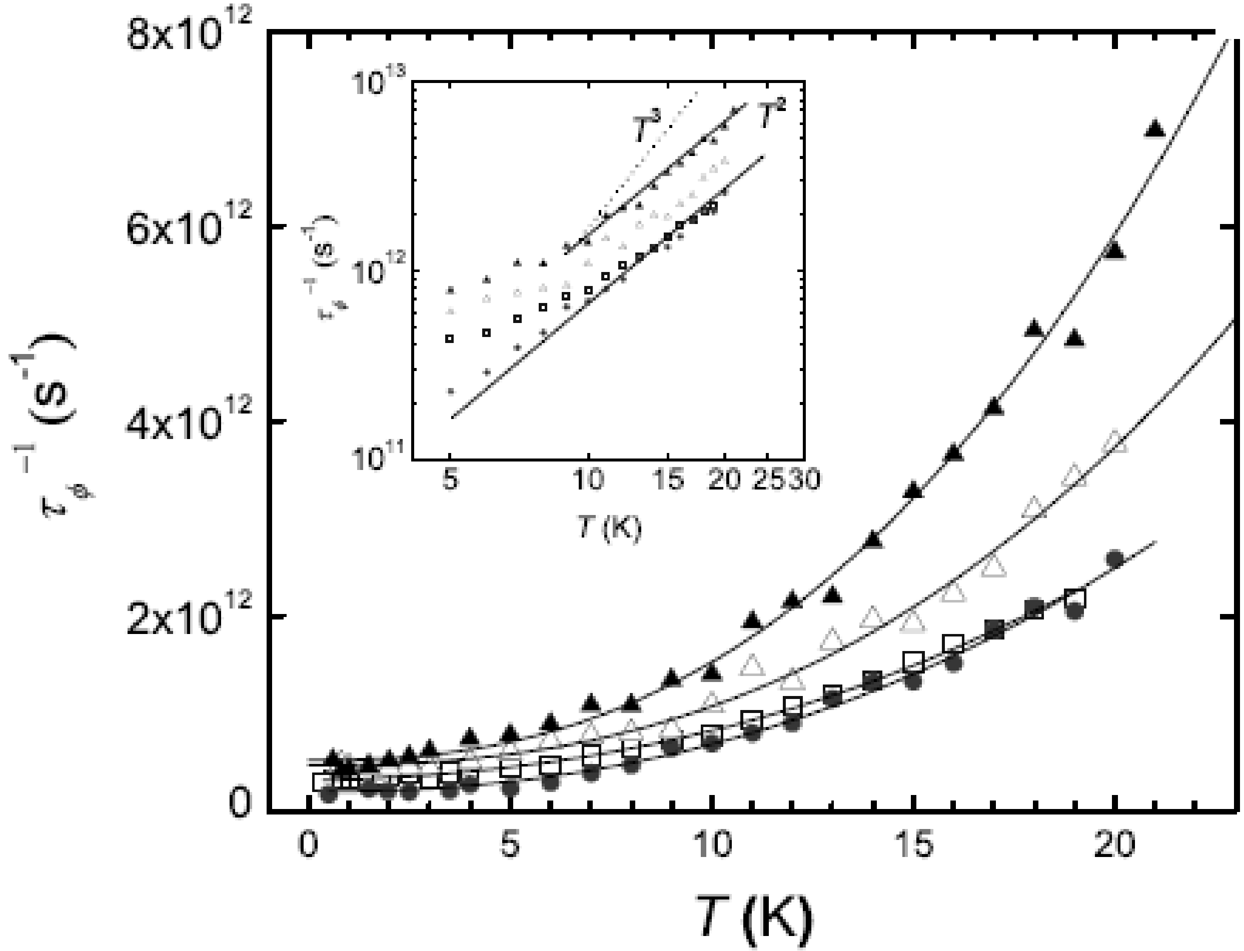
Pippard Model: Quasi-Ballistic Limit

qℓ>1

$$\frac{1}{\tau_{e-t.ph}} = \frac{3\pi^2 \beta_t T^2}{(p_F u_t)(p_F \ell)}$$



J.J. Lin et al. (2010)



AuPd nanowires: electron-phonon contribution to the electron dephasing rate = $A_{e\text{-}ph}T^2$

Coefficient $A_{e\text{-}ph}$ is in good agreement with the Pippard theory.

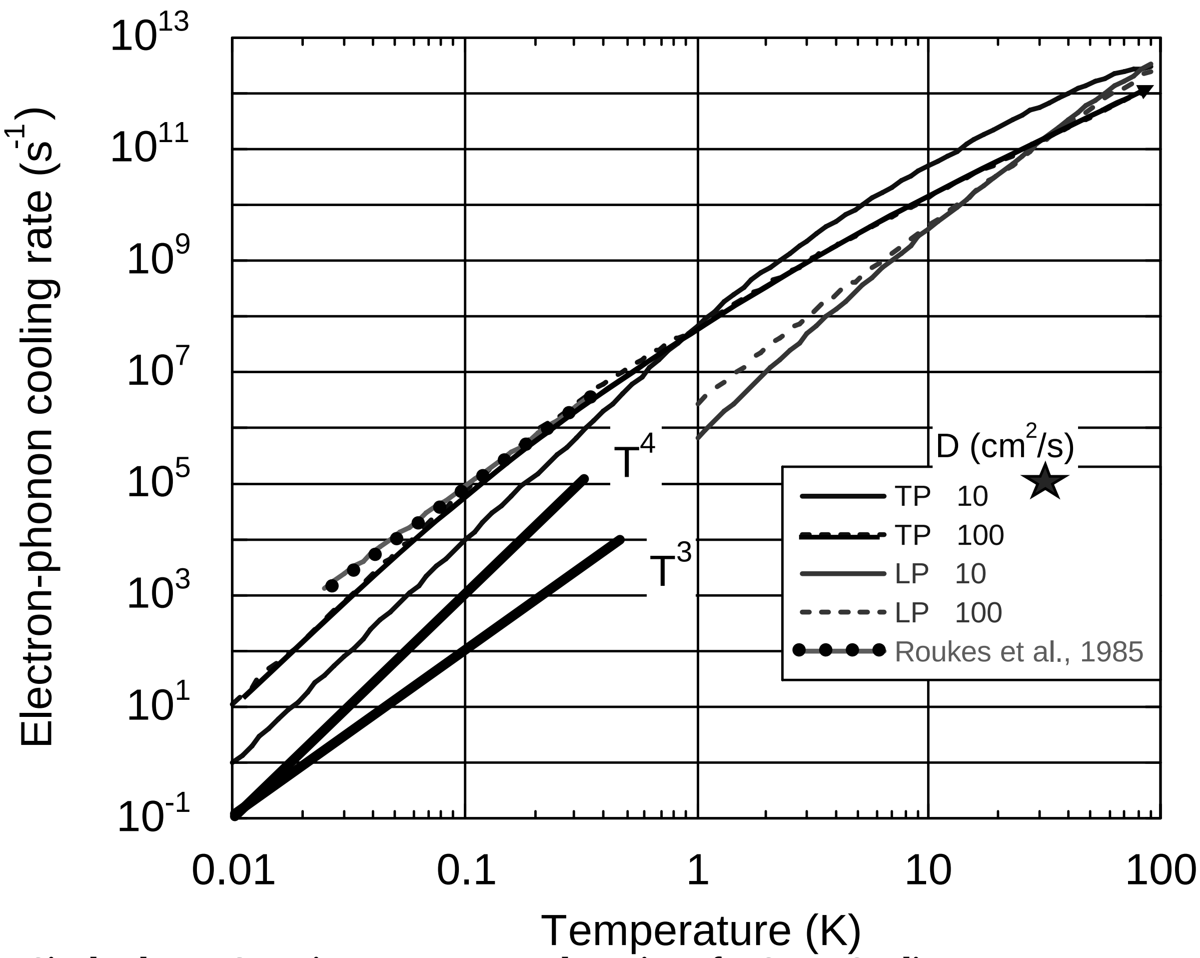
Intermediate Regime: *qℓ ~ 1*

Sergeev & Karasik, *Physica B*, 316-317, 328 (2002)

$$\frac{1}{\tau_{e-t.ph}} = \frac{3\pi^2 \beta_t T^2}{(p_F u_t)(p_F \ell)}$$

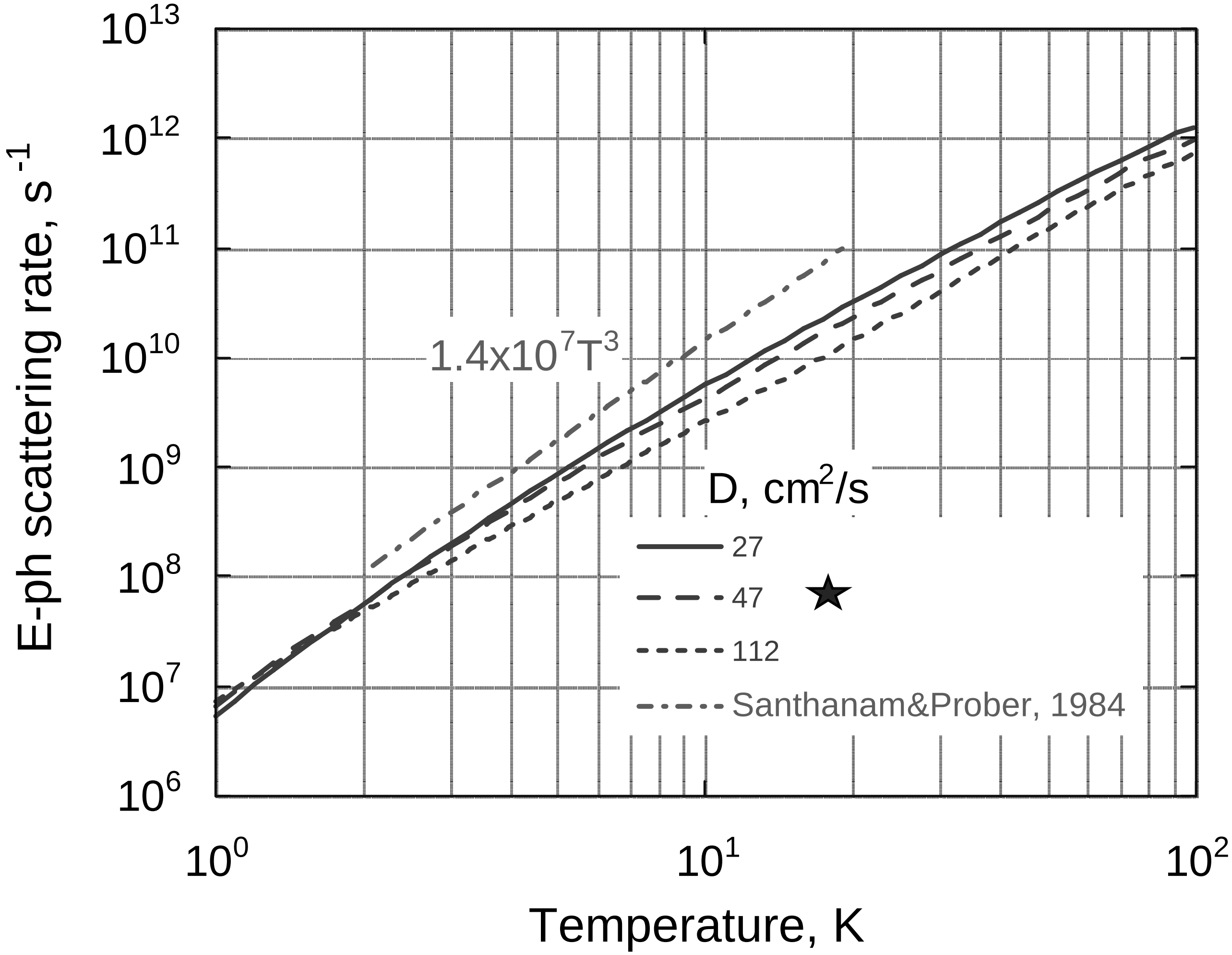
$$\frac{1}{\tau_{e-t.ph}} = \frac{3\pi^4 \beta_t}{10} \frac{p_F \ell T^4}{(p_F u_t)^3}$$

e-ph relaxation in Cu, Roukes (1985)



- **At low temperatures TP dominate even in "clean" films.**
- **Below 1 K, the LP contribution is 100 times weaker than the TP contribution.**
- **The observed T^3-dependence is a crossover between the clean limit TP T^2-law and the dirty limit T^4-law. The ℓ-dependence is almost absent in this region.**

e-ph relaxation in Al, Santhanam & Prober (1984)

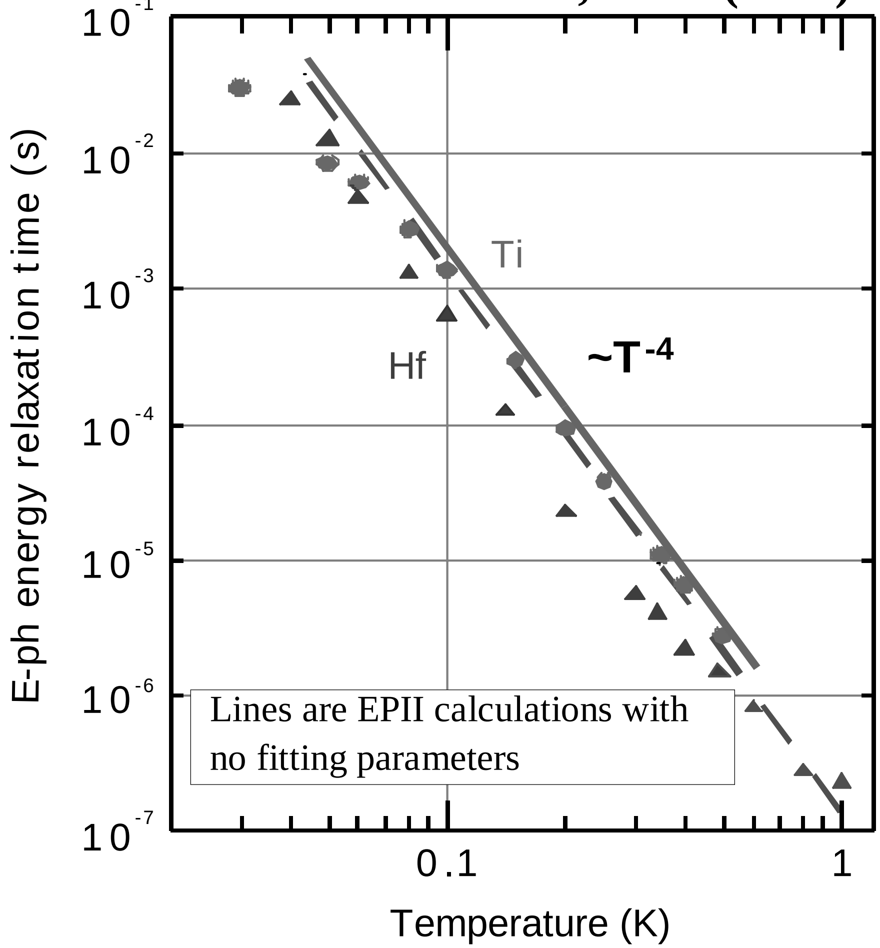


Diffusive Regime: $q\ell$ <1

e-ph relaxation in Hf and Ti

$$\frac{1}{\tau_{e-t.ph}} = \frac{3\pi^4\beta_t}{10}\frac{p_F\ell T^4}{(p_F u_t)^3}$$

M.E.Gershenson et al., APL (2001)



Magnetron sputtered films on <u>sapphire</u> substrates (acoustic impedances of sapphire and Hf (Ti) are very close)

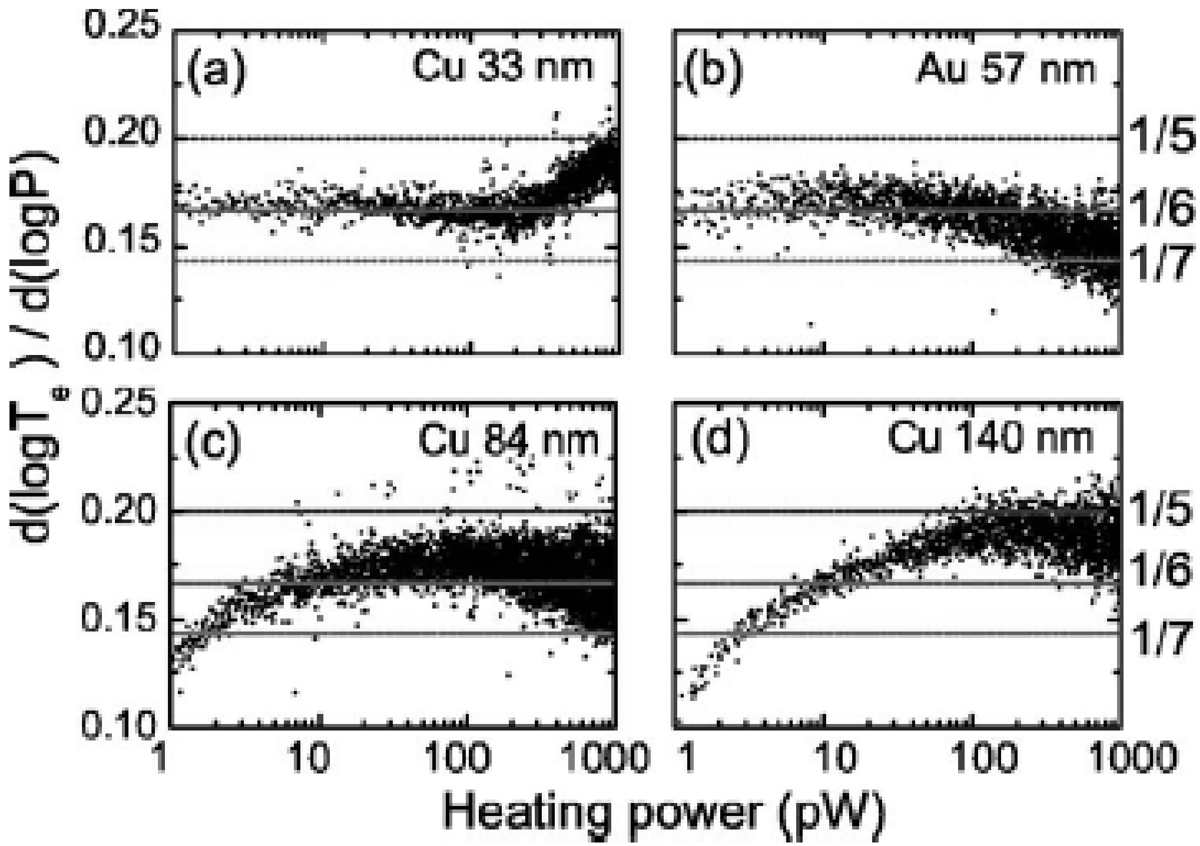
Hf
d = 25 nm
R = 38 Ω
T_c = 0.3-0.48 K
D = 1.5 cm^2/s

Ti
d = 20 nm
R = 15 Ω
T_c = 0.43 K
D = 2.5 cm^2/s

Diffusive regime: Cu and Au films
Karvonen, Taskinen, and Maasilta (2005)

$$\frac{1}{\tau_{e-t.ph}} = \frac{3\pi^4\beta_t}{10}\frac{p_F \ell T^4}{(p_F u_t)^3}$$

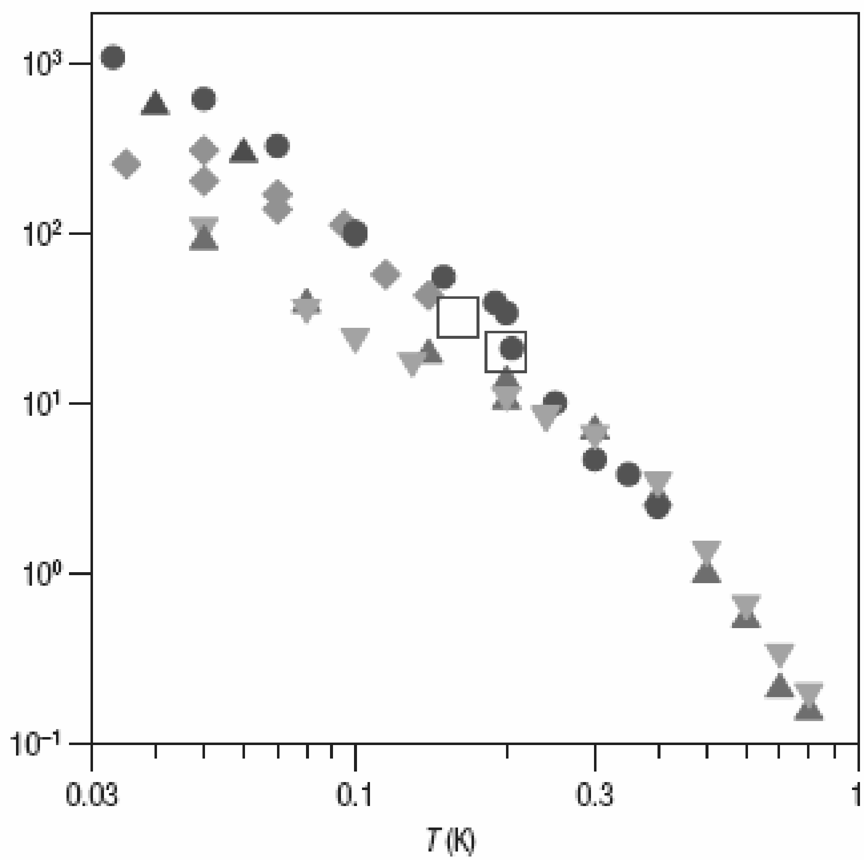
We have used symmetric normal metal-insulator-superconductor NIS tunnel junction pairs for ultrasensitive thermometry in the temperature range 50–700 mK.
Joule heating the electron gas and measuring the electron temperature, we show that the electron-phonon scattering rate in the simplest noble-metal disordered thin films (Cu, Au) follows a T^4 temperature dependence leading to a stronger decoupling of the electron gas from the lattice at the lowest temperatures.

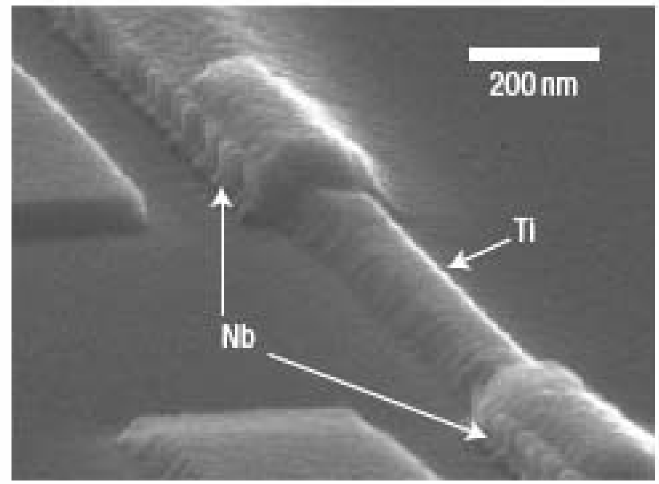


n = 1/p -2

Problems with the Pippard model in the diffusive limit

Ti nano_HEBs on <u>silicon</u> substrate
Wei et al. (2008)





$$\tau_{e-ph}^{-1} \propto T^4 \ , T > 0.3\text{K}$$
$$\tau_{e-ph}^{-1} \propto T^2 \ , T < 0.3\text{K}$$

Relaxation in alloys

The Pippard model fails to describe the e-ph relaxation in alloys.

- Lin JJ, Wu CY, Europhys. Lett 29, 141 (1995) ;

"Disorder dependence of the electron-phonon scattering time in bulk TiAl alloys" , T^2-dependence was associated with different vibrations of Ti and Al atoms.

- J.J. Lin et al. (1996, 1998, 1999):

$Ti_{1-x}Sn_x$ films with $\rho_0 > 100\ \mu\Omega$ cm ($k_F\, l$~5) show $1/\tau_{e\text{-}ph} \sim T^2/l$

$Ti_{1-x}Sn_x$ films with $\rho_0 < 70$cm $\mu\Omega$ ($k_F\, l$~7-10) show $1/\tau_{e\text{-}ph} \sim T^3/l$

$Ti_{73}Al_{27}$ films with $\rho_0 = 225\ \mu\Omega$ cm ($k_F l$~3-5) show $1/\tau_{e\text{-}ph} \sim T^2$

- Any complications of the Pippard model (non-Born scattering from impurities, specific impurity positions, etc.) anyway lead to the Pippard ineffectiveness condition.
- The Pippard model = Comoving frame of reference =

  = Tsuneto transformation = Deformation potential concept
- To obtain something different from the Pippard ineffectiveness one should go beyond the Pippard model.

  Sergeev-Mitin model:
- Any scatterers that vibrate with amplitude different from amplitude of host atoms, for example, static electron scatterers
- We understand well the quantum interference kinetics,

  but we still do not understand acoustics, i.e. vibrations of defects, defect clusters, grain boundaries…

SM model for quasi-static scatterers:

$$\frac{1}{\tau_{e-ph}} = \frac{\pi^4 T^4 l}{5 p_F}\left[\frac{\beta_l}{u_l^3} + \left(1 - \frac{l}{L}\right)\frac{3\beta_t}{2u_t^3}\right] + \frac{3\pi^2 T^2}{2 p_F L}\left[\frac{\beta_l}{u_l} + \left(1 - \frac{l}{L}\right)\frac{2\beta_t}{u_t}\right]$$

In general,

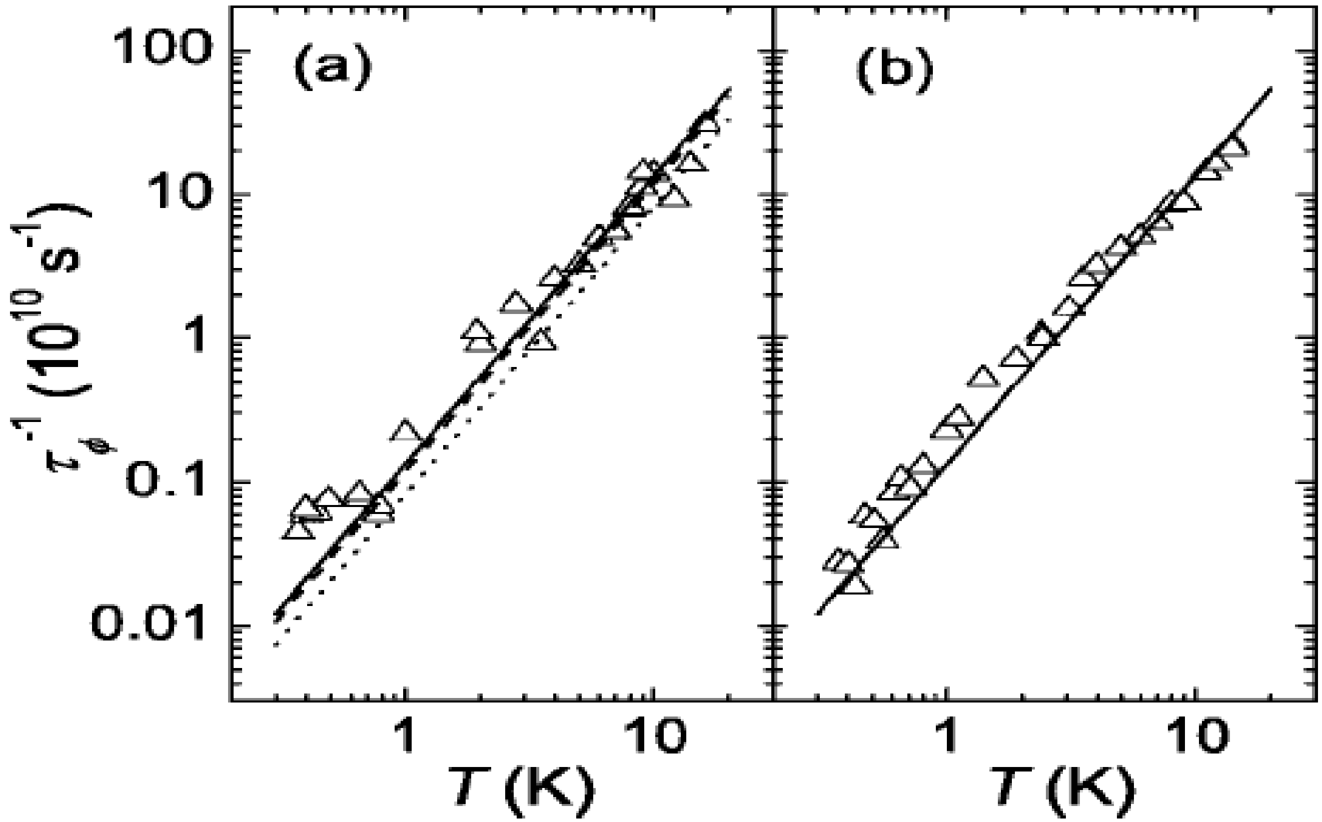
$$\frac{1}{\tau_{e-ph}} \approx \frac{T^3 (q_T \ell)}{(p_F u)^2} + g\,\frac{T^3}{(p_F u)^2 (q_T \ell)} = \frac{T^4 \ell}{u (p_F u)^2} + g\,\frac{T^2 u}{(p_F u)^2 \ell}$$

Experiments (2005): T^2-dependence has been observed in

- Biswas D, Meikap AK, Chattopadhyay SK, et al. — VPd
- Stolovits A, Sherman A, Kremer RK, et al. — NbTe
- Du J, Li ZQ, Lin JJ, et al.: — $Sb(SiO_2)$
- Biswas D, Meikap AK, Chattopadhyay SK, et al. : — TiVAl
- Ceder R, Agam O, Ovadyahu Z: — (InO)Au
- Biswas D, Melkap AK, Chattopadhyay SK, et al.: — ZrSn

SM model predicts the maximum value of the relaxation rate

$$\frac{1}{\tau_{e-ph}} = \frac{3\pi^2 \beta_t T^2}{4(p_F \ell)(p_F u_t)}$$



J. J. Lin et. al, Electron-phonon dephasing time due to the quasistatic scattering potential in metallic glass CuZrAl, *Phys. Rev. B* **74**, 172201, (2006).

Effects of phonon dimensionality (?)

DiTusa et al. PRL (1992),
"Role of the phonon dimensionality on the electron phonon scattering rate,"
10 – 100 nm thick, suspended and supported CuCr films at 0.5-10K
Conclusion:
We observe that quantization of the phonon spectra required by the sample dimensions has no effect on the magnitude and the temperature dependence of the relaxation rate.

Karvonen & Maasilta, PRL (2007)
Cu films on suspended silicon nitride membranes (thicknesses from 30 to 750 nm)
Conclusion:
At $T < 0.5$ K, the thinnest membranes can have a factor 2–3 strengthening effect, whereas at $T > 0.5$ the thermal relaxation from membranes can be an order of magnitude weaker compared to bulk samples.

Conclusions

• Disorder drastically changes the e-ph relaxation rate;

• In the quasi-ballistic limit, disorder increases the e-ph relaxation (no experimental evidence yet, subject for PRL paper);

• In the diffusive limit e-ph interaction has the interference origin:

• To increase the e-ph relaxation rate (constructive interference, SM model), one should use superconducting alloys, films with defects on substrates with significant acoustic mismatch with respect to the film;

• To realize Pippard's ineffectiveness condition,
one can try good films of ordinary superconductors with "natural defects" on substrates that match to the films.

Detector performance in terms of the quasiparticle number, *N*

• **The sensitivity of the sensor is limited by *the absolute fluctuations* of the equilibrium number of quasiparticles, N_{eq}.**

• **To be registered, the absorbed quantum should generate more than $N_{eq}^{1/2}$ quasiparticles,**

$$\delta N = h\nu / \varepsilon^{*} \geq \sqrt{N_{eq}} \ .$$

• **Response to classical field,**

$$\delta N = P\tau_{l} / \varepsilon^{*} \geq \sqrt{N_{eq}} \ ,$$

where *P* is the radiation power absorbed by the sensor,
τ_{ℓ} is the quasiparticle lifetime = the sensor's operating time.

• **Detector is characterized by <u>the noise equivalent power</u>**

$$NEP = P_{\min} / \sqrt{\Delta\omega} \approx \varepsilon^{*} \sqrt{N_{eq} / \tau_{l}} \ , \ \Delta\omega \approx 1/\tau_{l} \ .$$

Hot-Electron Nanobolometers:

Combination of ultra-small heat capacity with unparallel thermal isolation

State-of-the art performance @ 40 mk

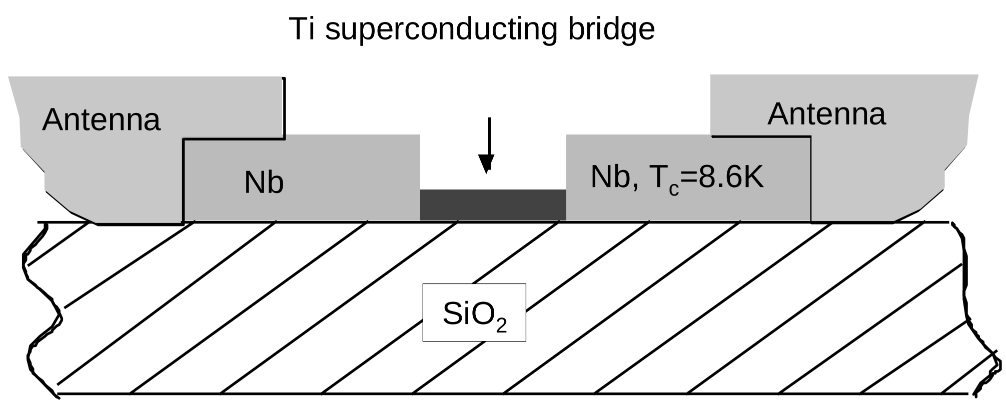
Electron heat capacity: $C_e = 3{\cdot}10^{-20}$ J/K.

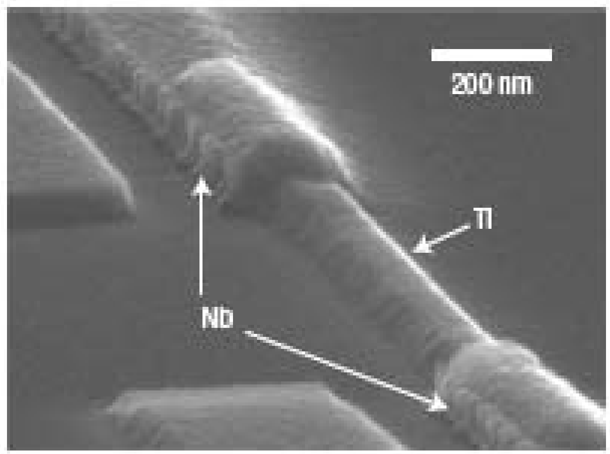
N ~ 1000

Electron-phonon thermal conductance: $G_{e\text{-}ph} = 0.6{\cdot}10^{-16}$ W/K

Relaxation (cooling time): $\tau_{e\text{-}ph} = 5{\cdot}10^{-4}$ s

(J. Wei et al, Nature Nanotechnology 3, 496, 2008)





Hierarchy of Relaxation Processes

- Thermolization of the electron subsystem (forming of the electron distribution with the nonequilibrium electron temperature) takes place at the time scale of the order of the electron-electron scattering time: $\tau_{th} \sim \tau_{e\text{-}e}$.

- Electron cooling is determined by the electron-phonon interaction, i.e. the corresponding time scale is $\tau_{e\text{-}ph}$. In disordered films $\tau_{e\text{-}e} \sim T$ is substantially shorter than $\tau_{e\text{-}ph}$ even at helium temperatures (electron heating). At subKelvin temperatures the ratio of $\tau_{e\text{-}ph}$ to $\tau_{e\text{-}e}$ significantly increases.

- Both $\tau_{e\text{-}e}$ and $\tau_{e\text{-}ph}$ do not directly depend on the device volume.*

- The electron-phonon thermal conductance, $G_{e\text{-}ph} = C_e/\tau_{e\text{-}ph}$, is proportional to the electron heat capacity C_e, or to the number of "classical" quasiparticles in the sensor volume: $N = C_e/(3/2\ kT)$.

- Because all quasiparticles cool down within the time scale of $\tau_{e\text{-}ph}$, the characteristic time between single electron-phonon scattering events is

$$\tau^* \sim \tau_{e\text{-}ph}/N$$

Formal proof

- n=1/τ* - number of electron-phonon scattering events in the volume V per second - is given by

$$n = V \int \frac{d\vec{q}}{(2\pi)^3} I_{ph-e}(\vec{q})$$

where V is the volume, I_{ph-e} is the phonon-electron collision integral.

- Integrating we get

$$n = \frac{1}{4\pi} \left(\frac{k}{\hbar} \right)^4 \frac{VT^4}{u^2 v_F} F(q_T \ell)$$

- In the Pippard model

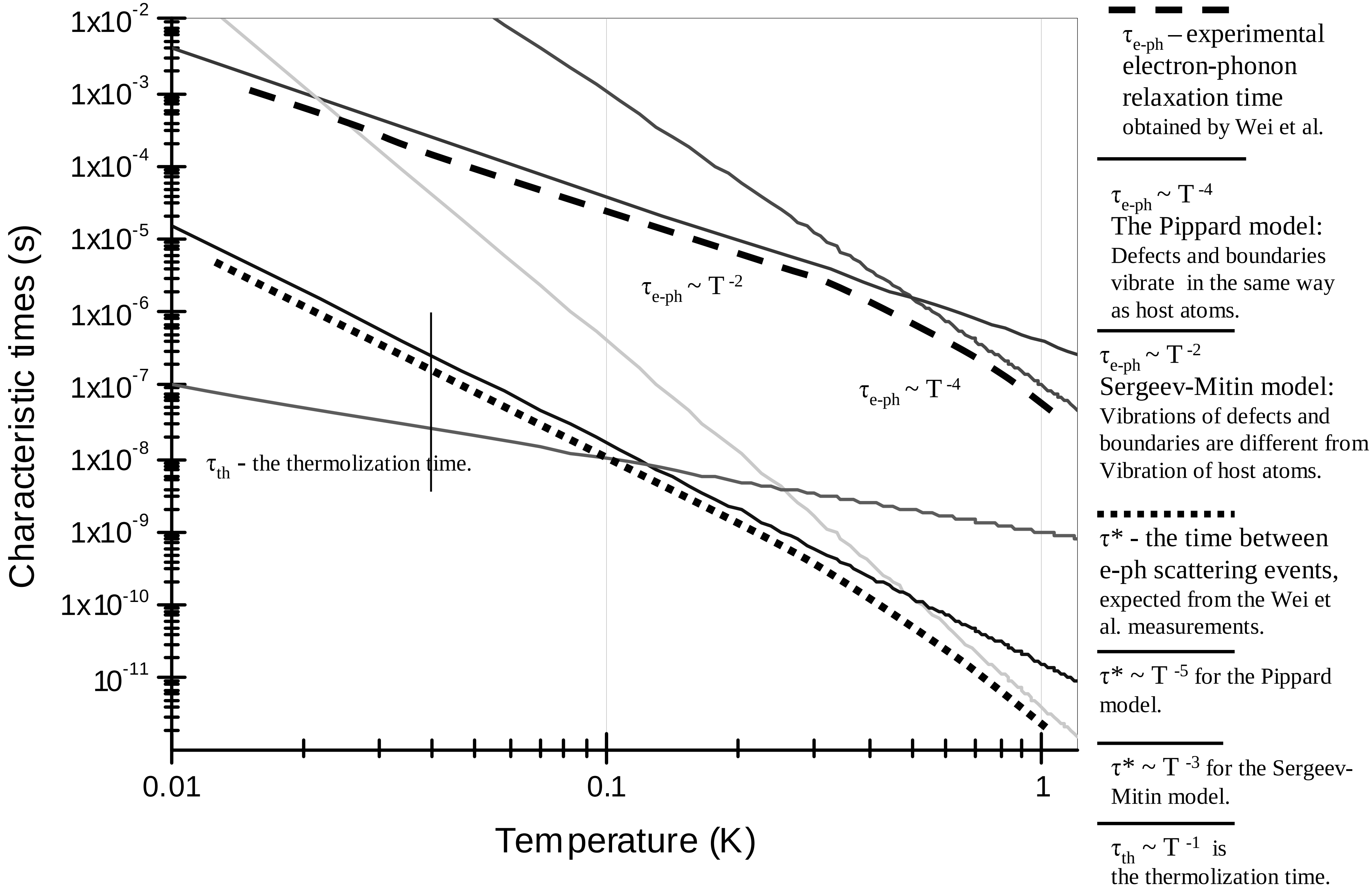
$$F(q_T \ell) = \frac{kT\ell}{\hbar u};$$

- In the Sergeev-Mitin model

where b describes the difference in vibration of electron scatterers (defects & boundaries) and host atoms.

$$F(q_T \ell) = b \frac{\hbar u}{kT\ell},$$

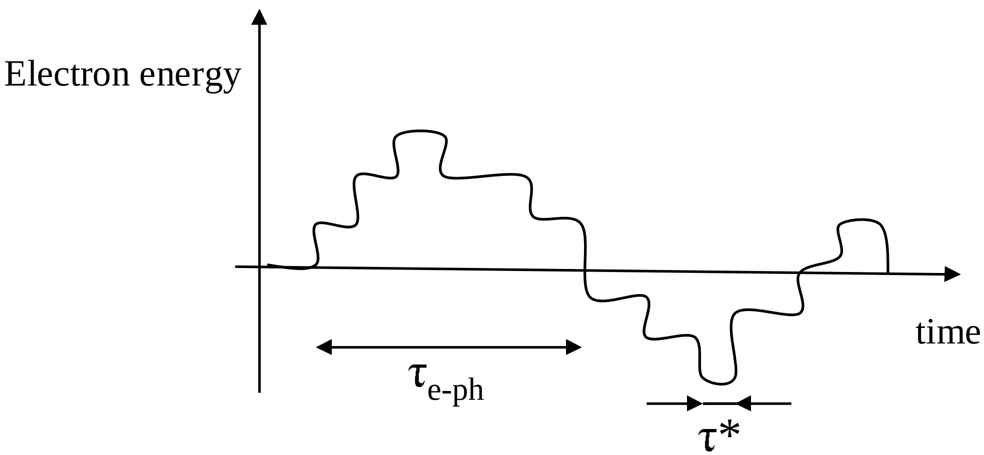
Characteristic Times for the State-of-the-Art Nanobolometers



Single Photon Counting Detectors, Keck Institute for Space Studies

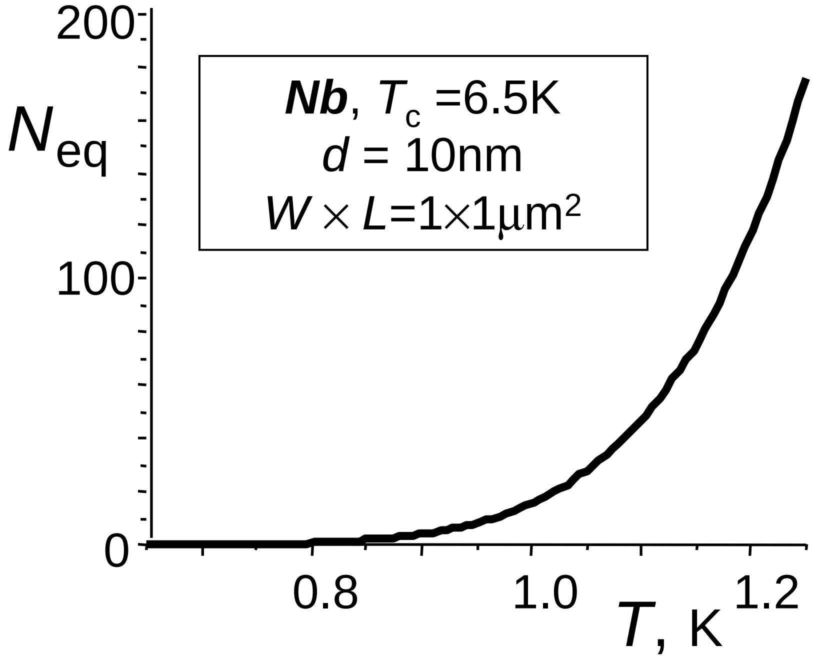
Can we observe single e-ph scattering events?

- To observe single electron-phonon scattering events, one should have $\tau_{th} << \tau^*$.
- For the state-of-the art nanobolometers, this condition takes place below ~0.1K.
- In this case, between electron-phonon scattering events the electron subsystem is fully isolated from its surroundings.
- In every e-ph scattering, the energy of electron subsytem changes by ~*kT/N* i.e. the temperature changes by *T/N*.

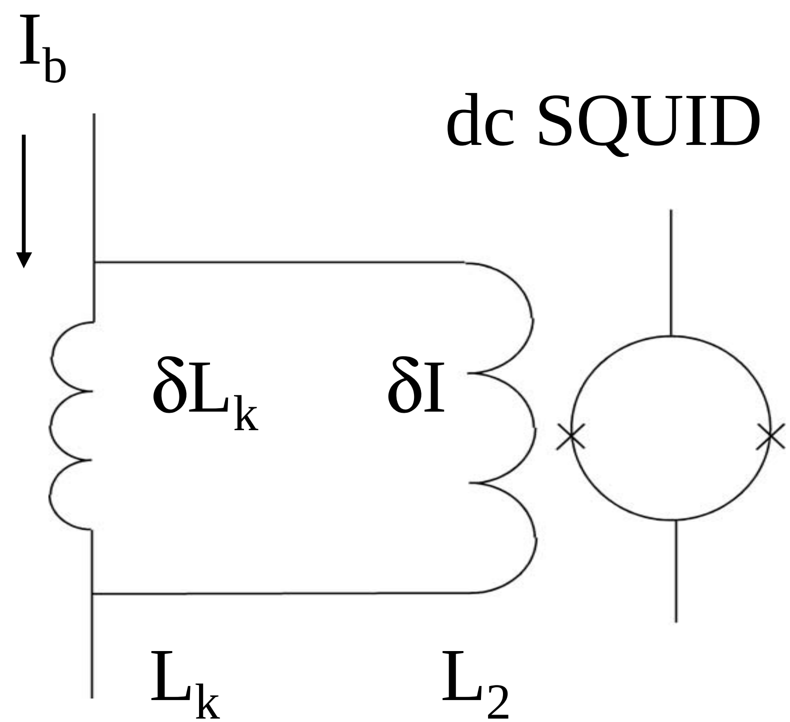


Kinetic-Inductance Detectors:

Small number of quasiparticles at helium temperatures



At $T << T_C$, a μm-size sample contains "mesoscopic" number of quasiparticles at helium temperatures.



An increase of the kinetic inductance caused by radiation decreases the current flowing through the SQUID coil producing detectable magnetic field.

The best KID performance is expected at $T << Tc$: $NEP \sim 10^{-19}$-10^{-20} W/Hz$^{1/2}$ at T = 1-1.5 K is limited by the generation-recombination noise.

Conclusions

- Small number of quasiparticles, *N*, and, therefore, use of nanostructures is the key issue for high performance of hot-electron nanobolometers and KI detectors.

- In nanosensors, the electron-phonon scattering events are separated by the characteristic time τ^*, which is $\sim \tau_{e\text{-}ph} / N$. Between e-ph scattering events the electron subsystem is fully isolated from its surroundings.

- With the state-of-the-art hot-electron nanobolometers, single electron-phonon scattering events can be observed at ~ 10mK.

- KI detectors are promising candidates for observation single e-ph scattering events.

Related papers

1. J. Wei, D. Olaya, B.S. Karasik, S.V. Pereverzev, A.V. Sergeev, and M.E. Gershenson, "Ultrasensitive hot-electron nanobolometers for terahertz astrophysics," Nature Nanotechnology, 3, 496 (2008).
2. B. Karasik and A. Sergeev, "Hot-electron superconducting photon counter", *IEEE Appl. Supercond*. **15**, 618 (2005).
3. A. Sergeev, V. Mitin, B. Karasic, and E. Gershenson: Superconducting nanosensors with mesoscopic number of quasiparticles. - *Physica E* **19**, 173 (2003).
4. A. Sergeev, V. Mitin and B. Karasic: Ultrasensitive kinetic-inductance detectors operating well below the superconducting transition. - *Appl. Phys. Lett*, **80**, 817 (2002).

Calculation details may be found in

Electron energy relaxation in semiconductor nanostructures

- A. Sergeev, M.Yu. Reizer, and V. Mitin: Deformation electron-phonon coupling in disordered semiconductors and nanostructures. – *Phys. Rev. Lett.*, **94**, 136602 (2005).

Electron energy relaxation in metals

- A. Sergeev and V. Mitin: Electron-phonon interaction in disordered conductors: Static and vibrating scattering potentials. - *Phys. Rev. B.* **61**, 6041-6047 (2000).
- A. Sergeev and V. Mitin: Breakdown of Pippard ineffectiveness condition for phonon-electron scattering in micro and nanostructures. - *Europhys. Lett.* **51**, 641-647 (2000).

Transport in semiconductor nanostructures

- A. Sergeev**,** M. Reizer, and V. Mitin: Effects of electron-electron and electron-phonon interactions in weakly disordered conductors and heterostructures. – *Phys. Rev. B.* **69**, 075310 (2004).

Electronic Kapitza conductance

A.V. Sergeev, Electronic Kapitza resistance due to inelastic electron-boundary scattering. - *Phys. Rev. B* **58,** R10199-10202 (1998).

Phonon drag thermopower

- A. Sergeev and V. Mitin: Effect of electronic disorder on phonon-drag thermopower. - *Phys. Rev. B.* **65**, 064301 (2002).

Applications

- B. Karasik and A. Sergeev, Hot-electron superconducting photon counter, - *IEEE Appl. Supercond* **15**, 618 (2005).