*RIT Detector Virtual Workshop*

# Photon Counting with InGaAsP Single Photon Avalanche Diodes

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**Mark A. Itzler**

Princeton Lightwave Inc.

Cranbury, NJ  08512  USA

*mitzler@princetonlightwave.com*

*www.princetonlightwave.com*

# Colleagues and Collaborators

**PLI Colleagues:**

**Xudong Jiang**
**Mark Entwistle**
**Mark Owens**
**Krystyna Slomkowski**
**Ketan Patel**
**Bora Onat**
**Sabbir Rangwala**

**External Colleagues & Collaborators:**

**Sergio Cova**.........*Politecnico di Milano*
**Alberto Tosi**
**Franco Zappa**
**Massimo Ghioni**

**Joe Campbell**.......*University of Virginia*
**Archie Holmes**

**Bill Farr**................*Jet Propulsion Laboratory*

**Mike Krainak**........*NASA/GSFC*

**Gary Smith**...........*MIT Lincoln Laboratories*
**Alex McIntosh**
**Simon Verghese**

**Alessandro Restelli**.........*NIST/Gaithersburg*
**Josh Bienfang**

**Zhiliang Yuan**................*Toshiba/UK*

**Naota Namekata**........*Nihon University*
**Shuichiro Inoue**

**Thomas Jennewein**........*University of Waterloo*
**Zhizhong Yan**

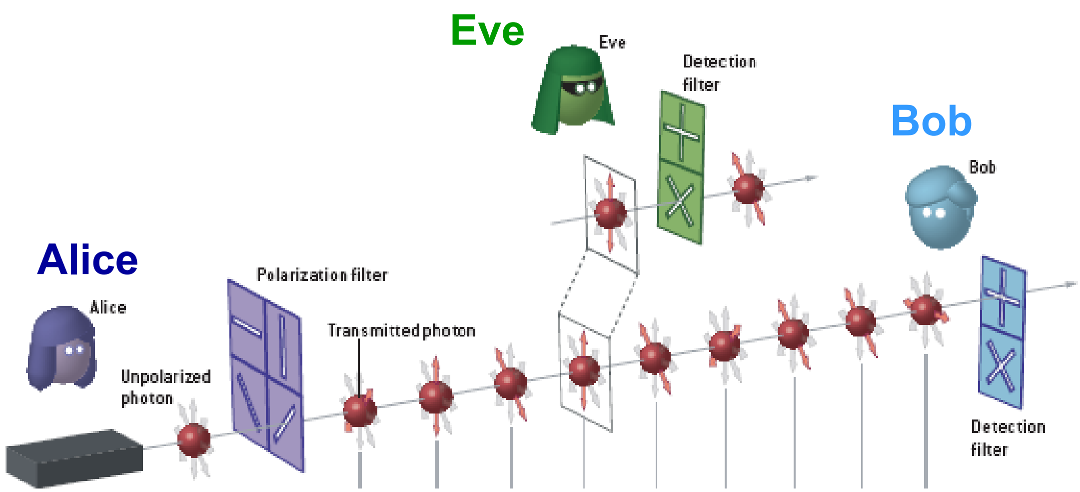
**Hugo Zbinden**........*University of Geneva*
**Tommaso Lunghi**

# Workshop Outline

- **Applications and drivers**
- **InGaAsP single photon avalanche diode (SPAD) fundamentals**
  - SPAD device design and performance parameters
- **High-rate photon counting with InGaAsP SPADs**
  - Challenges of high-rate counting: transients and afterpulsing
  - Progress in high-rate counting techniques
- **Free-running operation with self-quenching NFADs**
  - Integration of negative feedback
  - Self-quenching avalanche dynamics
- **Scaling to large format SPAD arrays**
  - Integration for focal plane arrays and FPA performance
- **Future prospects**
  - High-rate photon counting
  - "Solid state photomultipliers" based on NFADs
  - Photon number resolution with SPADs/NFADs
  - Further scaling and micropixellated arrays

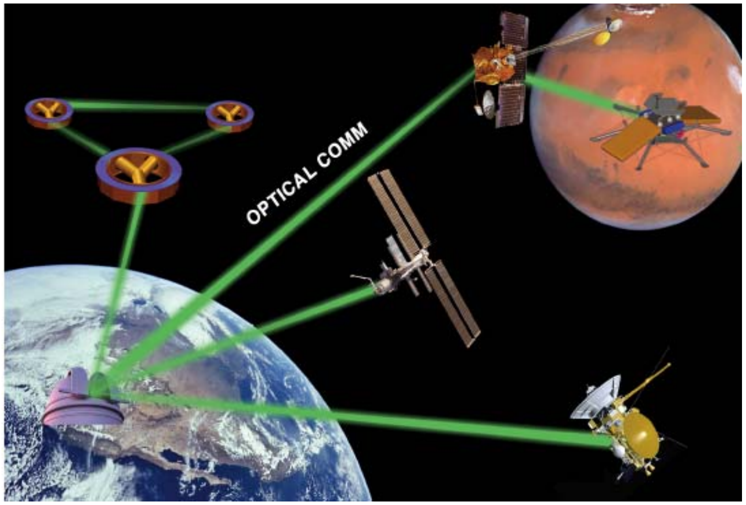
# High-rate photon counting SPAD applications

- **Exploiting quantum mechanical nature of photons**
  - quantum information processing (e.g., quantum cryptography and computing)




**Encryption keys using quantum properties of single photons**

- **Free-space communications and single-photon imaging**
  - long-range free-space optical communications
  - single-photon imaging with high photon arrival rate



# "Free-running" SPAD applications

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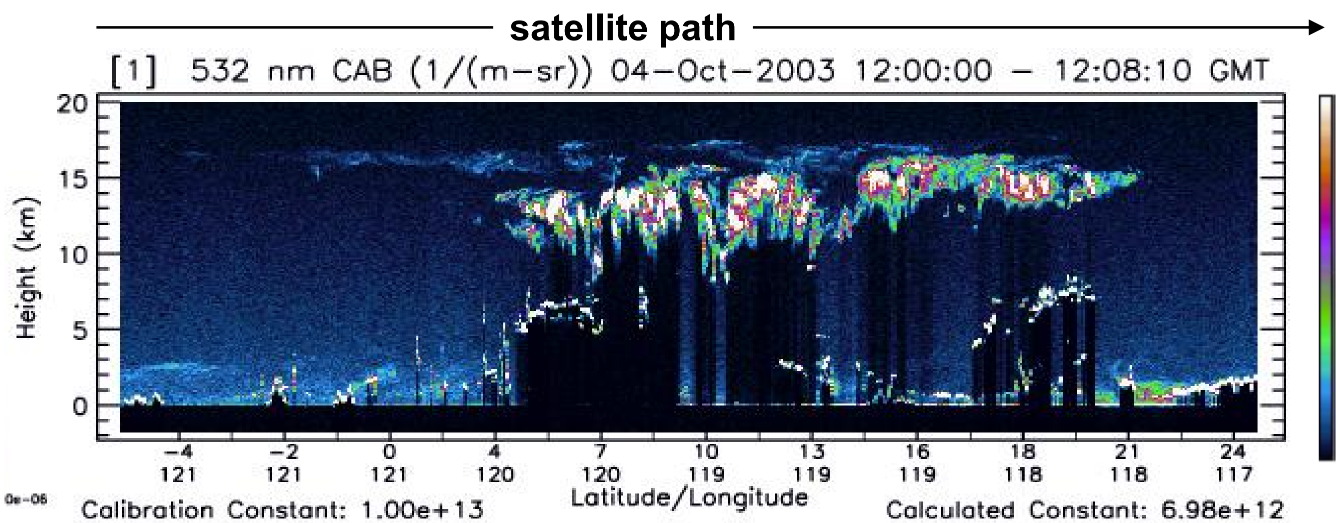
- **"Asynchronous" applications** (no knowledge of photon arrival time)
- **LIDAR measurements for earth science**
  - Distributed reflection from soft targets (clouds, aerosols)



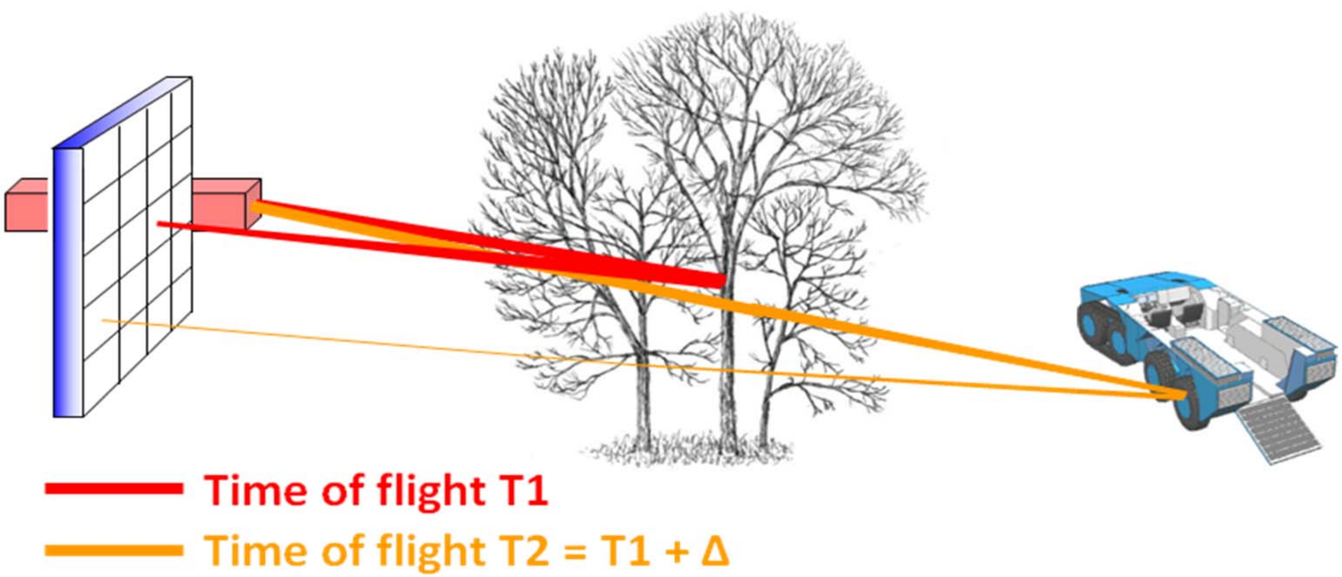

Atmospheric mapping by lidar along Earth's circumference

- **Flourescence measurements based on time-correlated SPC**
  - temporally random single photon emissions

# <u>Large-format arrays</u> required for imaging

- **Photon-starved low-light-level imaging applications**
  - Astronomy and astrophysics
  - Night vision
- **3-D LADAR (laser radar) imaging**
  - Perform independent LADAR measurement at every pixel of the imager
  - Time-of-flight information provides “depth” for generating 3-D point clouds

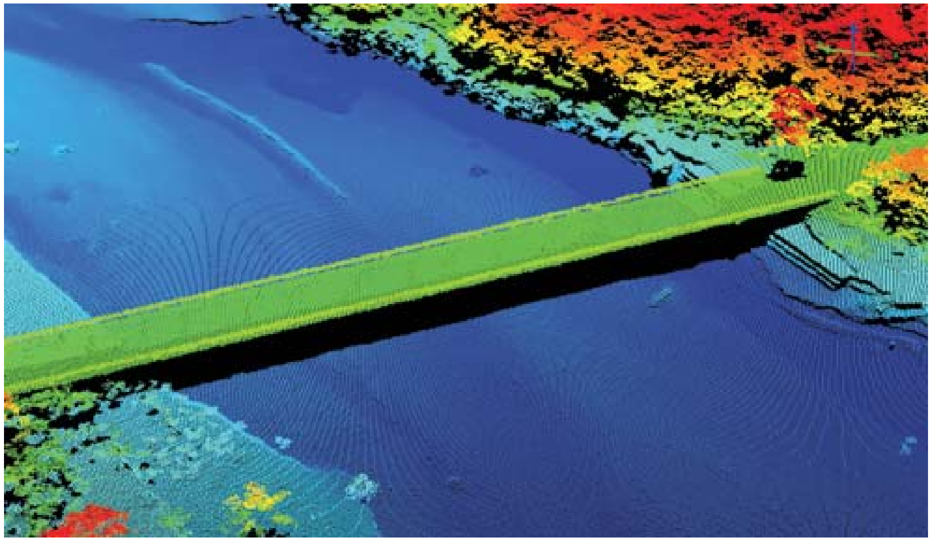
**3-D LADAR imaging concept**



# Example of 3-D LADAR mapping applications

- **Pioneering development of Geiger-mode APD 3-D LADAR at MIT Lincoln Lab**
- **Striking demonstrations of technology capability with MIT-LL ALIRT system**
  - **extensive mapping after Haiti earthquake in 2010**
  - **pair of 32 x 128 focal plane arrays scanned to obtain imagery**

**Assess trafficability (roads, bridges, etc.)**



**Terrain mapping, damage assessment, etc.**


http://www.ll.mit.edu/news/haitirelief.html

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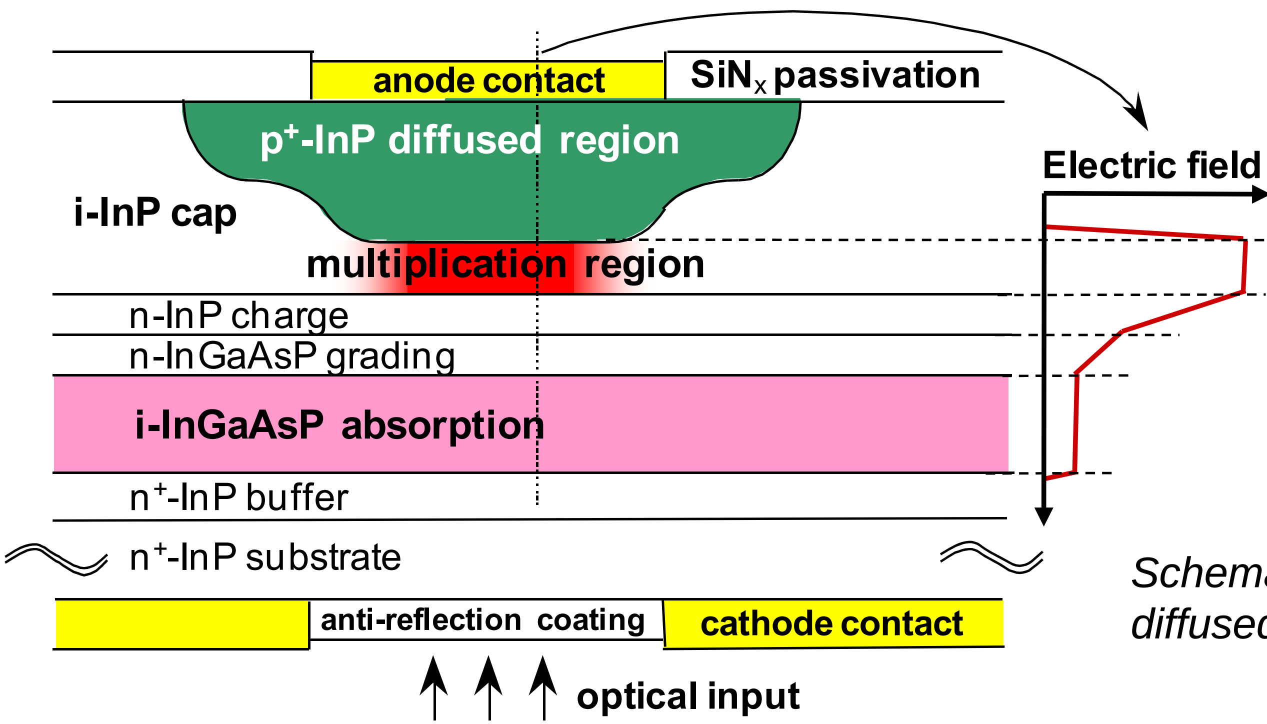
# Basic APD design platform

- **Low E-field in absorption region → collect carriers, but minimize noise**
- **High E-field in multiplication region → induce avalanche gain**

- **PLI has long history with planar-geometry InP/InGaAs APDs**
  - Stable and reliable buried p-n junction → very high yield and uniformity
  - Widespread deployment in telecom Rx as linear mode APD (LmAPD)
  - Re-engineered for single photon detection as Geiger mode APD (GmAPD)




*Schematic of InP/InGaAs diffused-junction APD*

# APD I-V Characteristics:  Linear & Geiger modes

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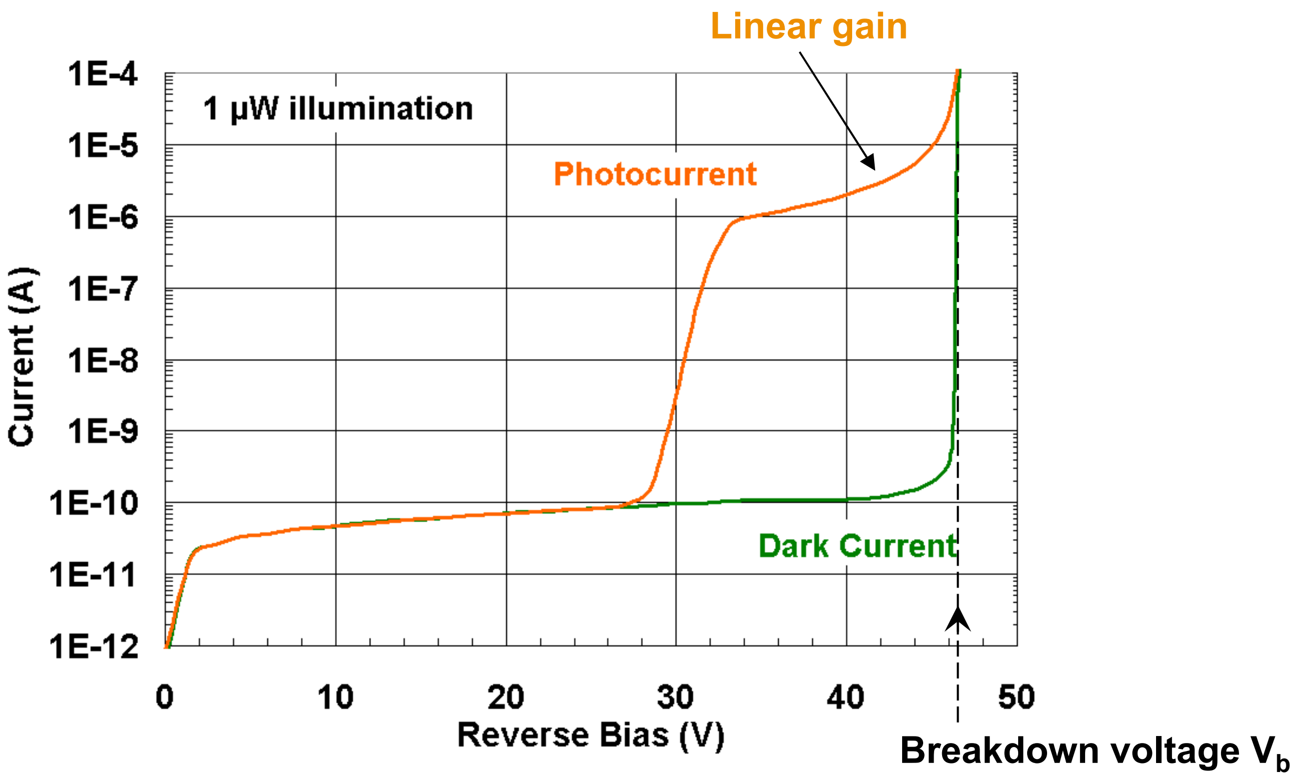
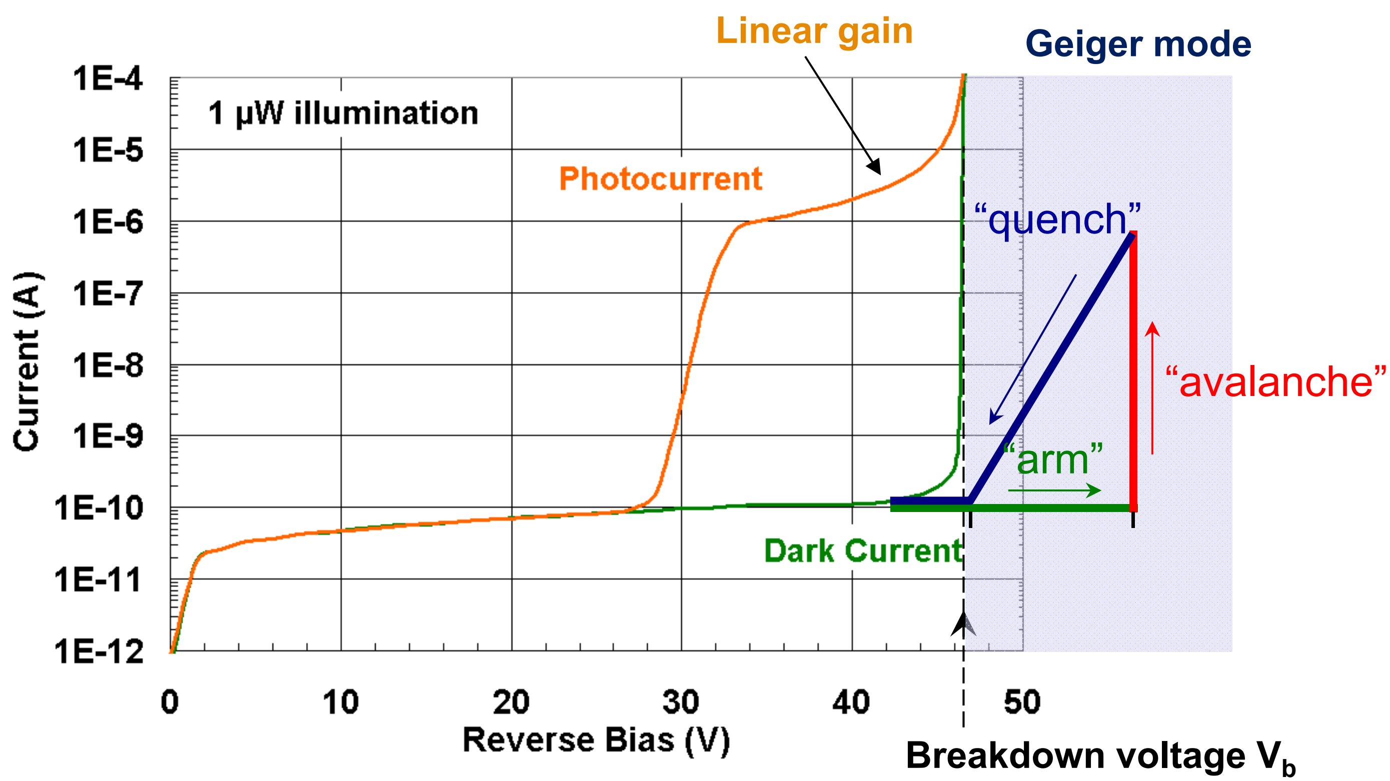
- **Linear mode performance defines behavior below breakdown voltage $V_b$**
  - Photocurrent below $V_b$ is proportional to input optical power → ANALOG



# APD I-V Characteristics: Linear & Geiger modes

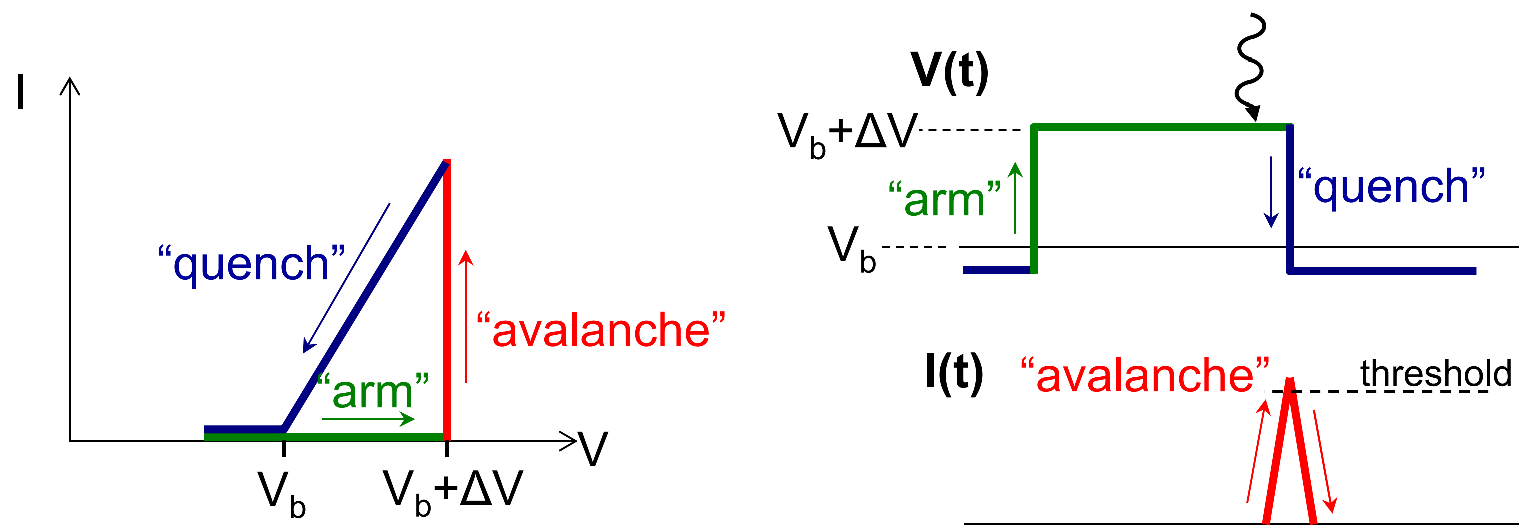
- **Linear mode performance defines behavior below breakdown voltage $V_b$**
  - Photocurrent below $V_b$ is proportional to input optical power → ANALOG
- **Geiger-mode performance has different device functionality**
  - Operation above $V_b$ can achieve self-sustaining avalanches → DIGITAL



# SPADs and Geiger-mode "effective gain"

- **SPAD generally viewed as a "photon-activated" switch**
  - Gain not strictly defined since avalanche can be self-sustaining
  - "Effective gain" dictated by combination of detector + circuit

  → **GmAPD "gain" ~ # of charges Q that flow per avalanche**



# Trends in single photon counting with APDs

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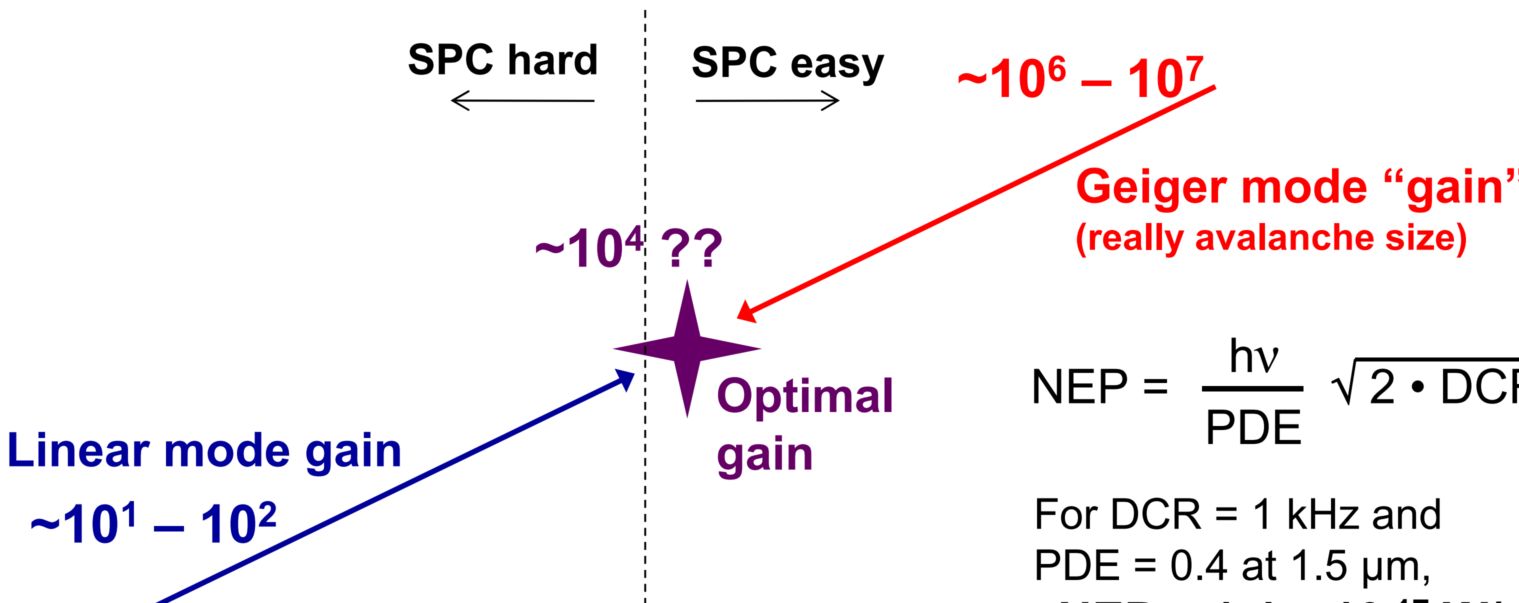
- **Linear mode APDs: need <u>more</u> gain**
  - Challenge to overcome noise of circuitry (analog)
- **Geiger mode APDs: need <u>less</u> gain**
  - Large charge flow Q is easy to detect (digital detection process is noiseless)
  - Challenge is to reduce Q to minimize limitations of afterpulsing and crosstalk
    - Present implementations limited to 1 detection event per pixel per frame




$$\text{NEP} = \frac{h\nu}{\text{PDE}}\sqrt{2 \bullet \text{DCR}}$$

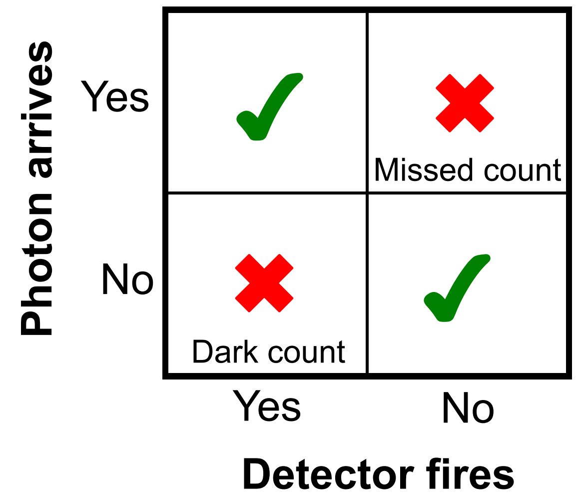
For DCR = 1 kHz and
PDE = 0.4 at 1.5 μm,
**NEP ~ 1.4 x $10^{-17}$ W/√Hz**

# Single photon detectors:  with & without photons...

## <u>Ideal detector</u>:

Always fires when a photon arrives

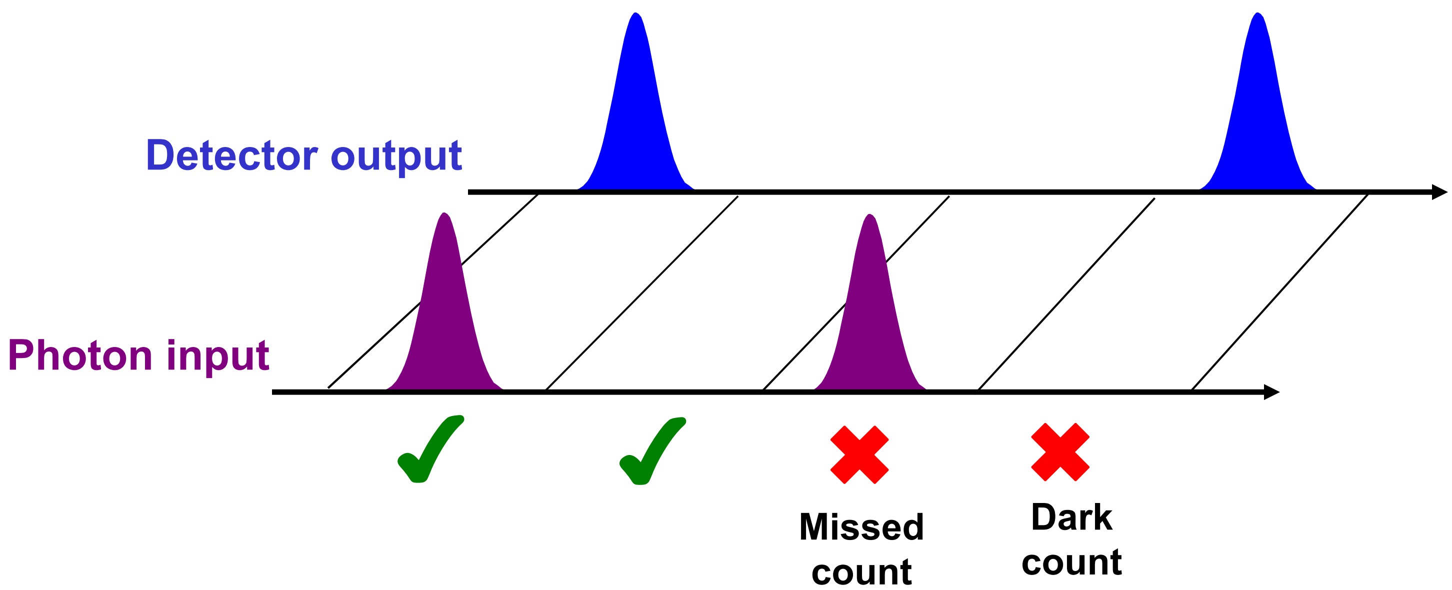
Never fires when a photon does not arrive




Photon Detection Efficiency (PDE):
probability that photon arrival causes detector to fire

Dark Count Rate (DCR):
probability that detector fires in absence of photon arrival

Prob(Missed count) = 1 - PDE



# SPAD Performance Parameters

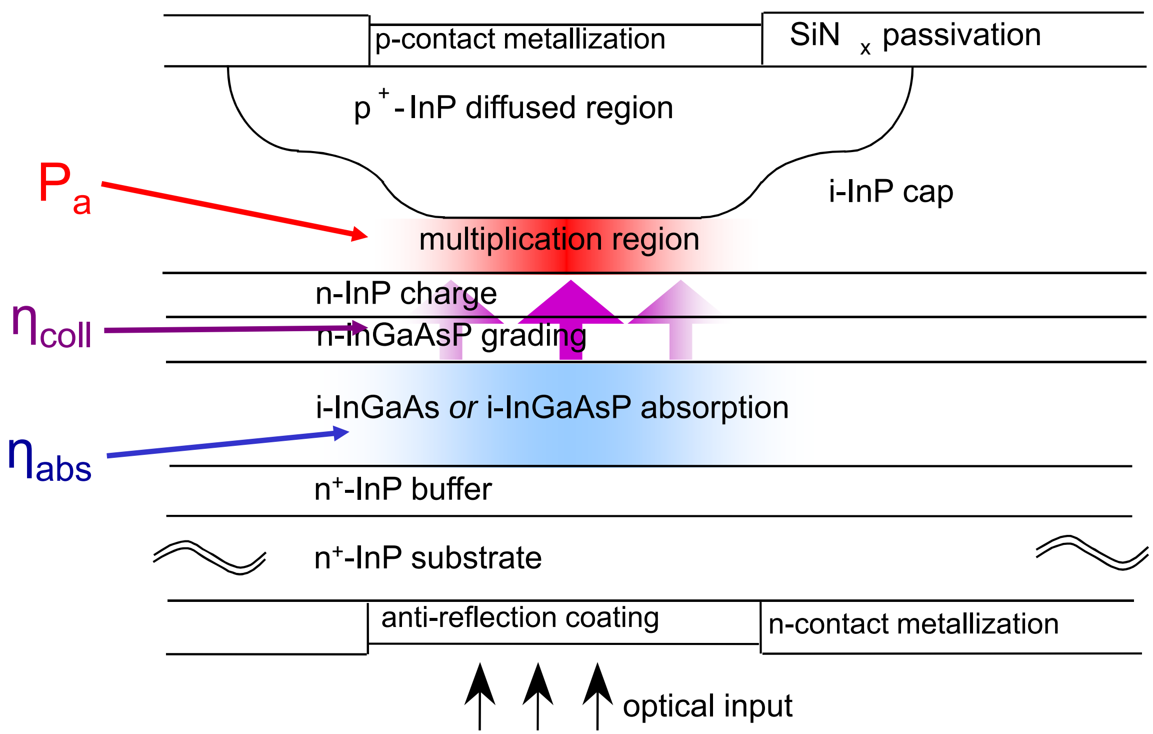
- **Photon detection efficiency (PDE):** probability of detecting incident photon
- **Dark count rate (DCR):** probability of "false" detection (no incident photon)
- **Timing jitter (TJ):** randomness in detection timing
- **Afterpulsing (AP):** increase in dark count rate following previous detection
  - Mitigated by sufficient "hold-off" time → ***BUT limits Counting Rate***

**Critical performance trade-offs must be managed**

- **Increase overbias: DE ☺ , TJ ☺ , DCR ☹**
- **Decrease temperature: DCR ☺ , AP ☹**

# Photon Detection Efficiency

- **Photon detection efficiency: PDE = $\eta_{abs} \times \eta_{coll} \times P_a$**
  - $\eta_{abs}$ : probability of photon absorption (i.e., quantum efficiency)
  - $\eta_{coll}$ : probability of carrier collection (injection to multiplication region)
  - $P_a$ : probability that collected carrier initiates detectable avalanche




**For well-designed SPADs:**

$P_a$ depends on bias; trade-off with DCR; improves with wider multiplier‡; >50%

$\eta_{coll}$ ~ 100%

$\eta_{abs}$ ~ 70 – 90%

‡ Ramirez, Hayat, et al., JQE 42, 137 (2006)

# Dark Count Rate

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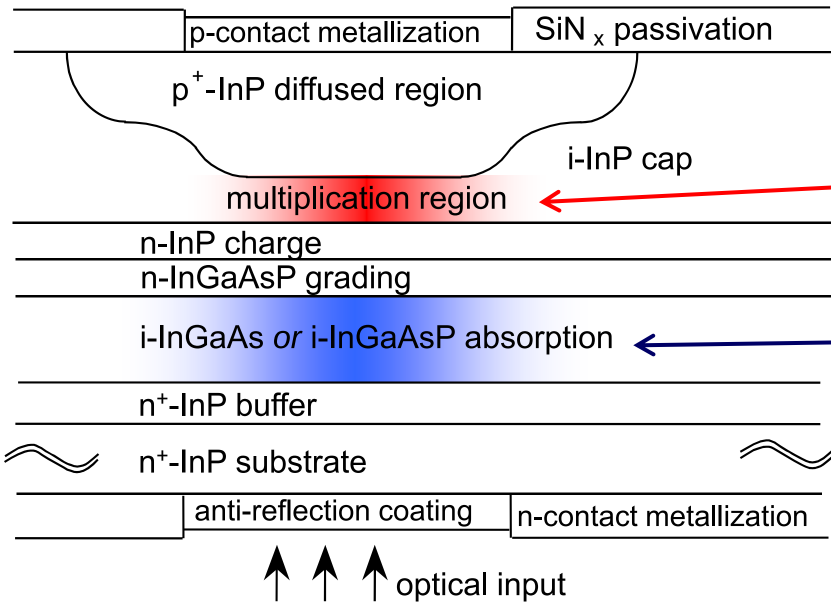
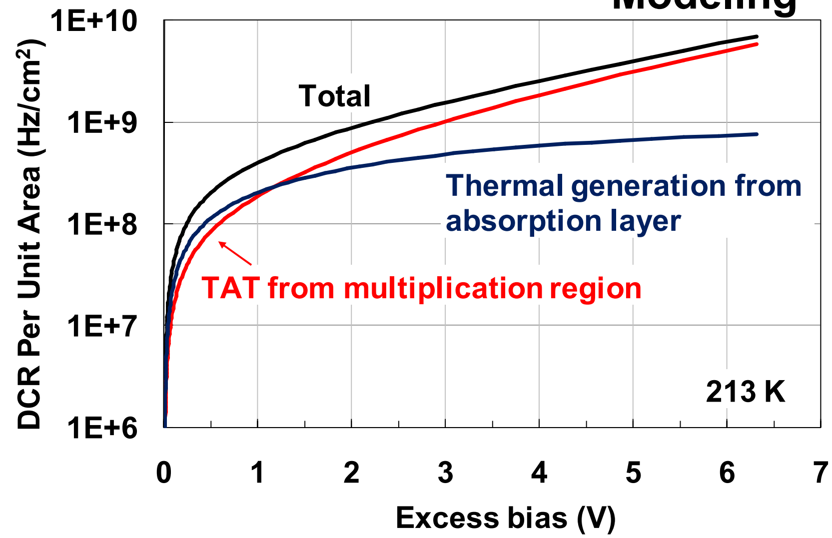
- **DCR dominated by two mechanisms in SPAD structure**




**Trap-assisted tunneling in large-bandgap (~1.35 eV) InP multiplier**

**Thermal generation in small-bandgap (~0.77 ev) InGaAs absorber**

**Modeling**

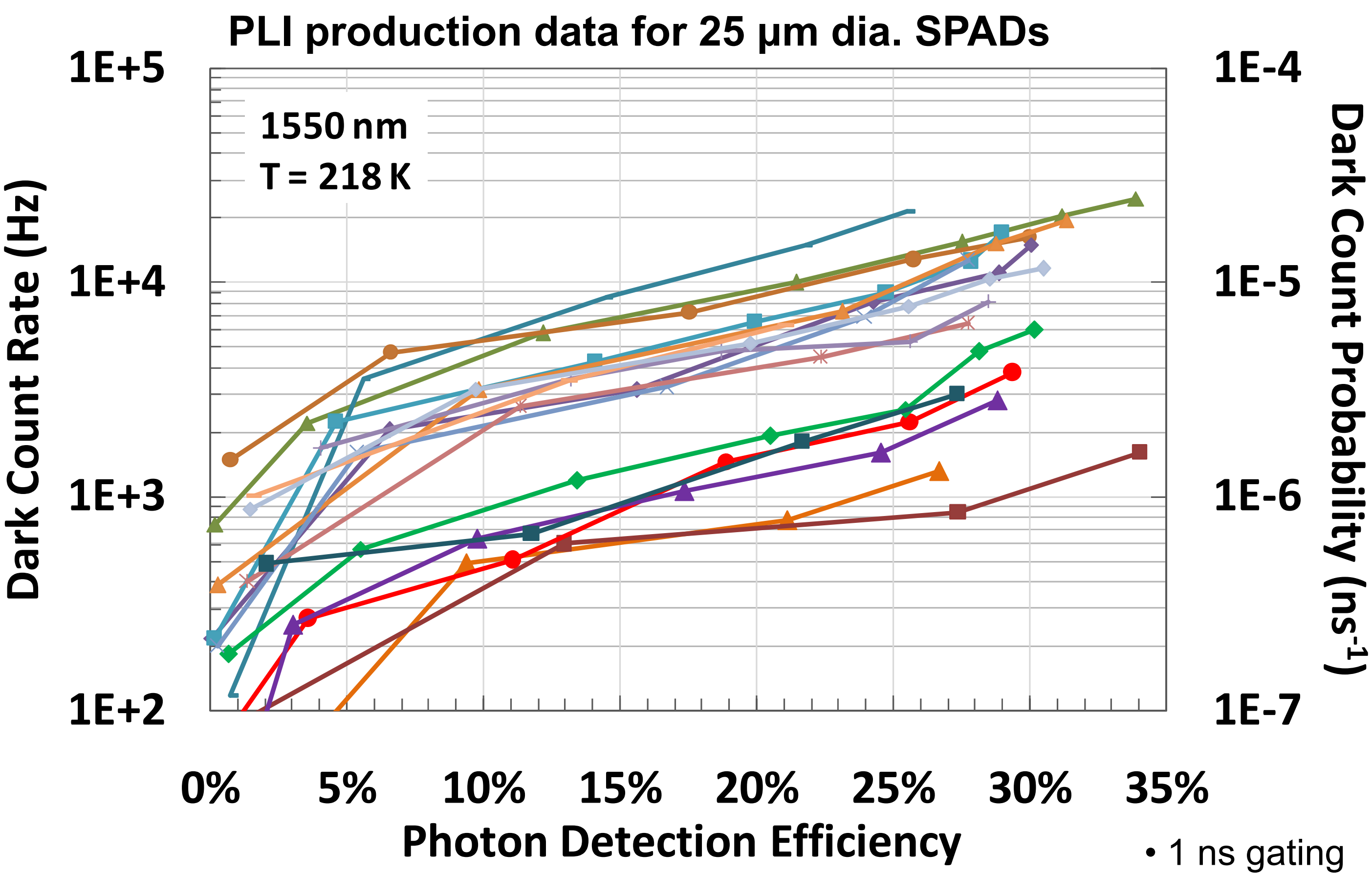



**Modeling provides insight into dependence of DCR mechanisms on temperature and bias**

Xiang, et al., Proc. SPIE **6771**, 677127 (2007)

# 1.5 µm SPAD DCR vs. PDE Performance

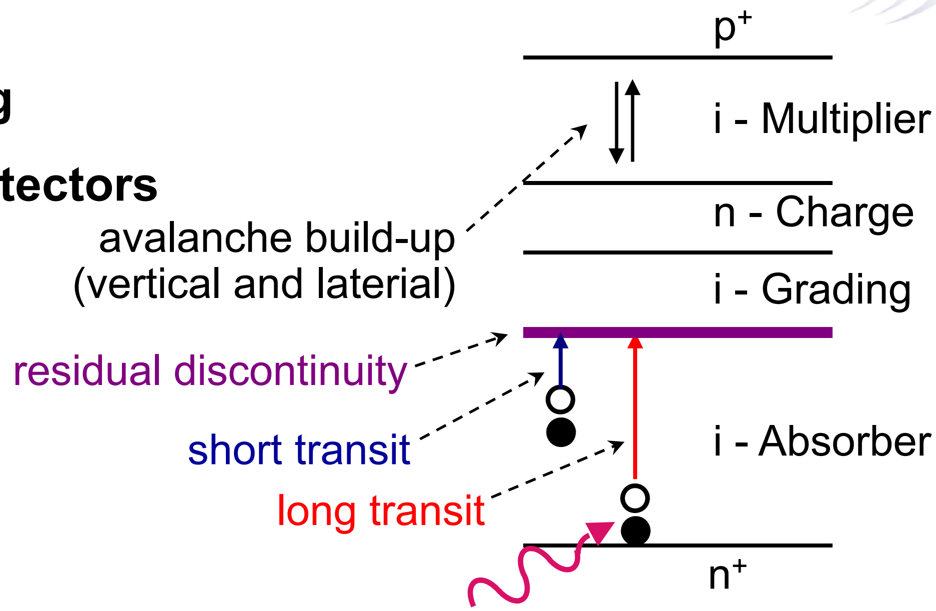
- **Fundamental trade-off: DCR and PDE both increase with bias**
- **State-of-the-art DCR: ~1 kHz at 20% PDE, ~2 kHz at 30% PDE**
  - Higher PDE accessible with larger bias

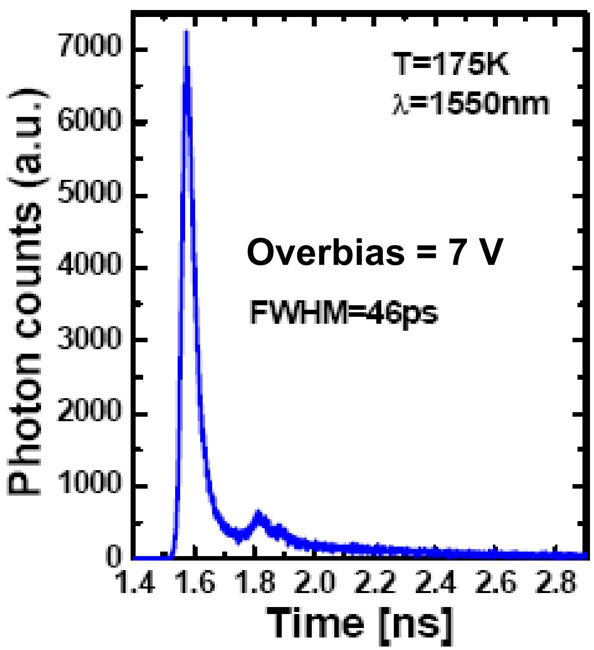



- 1 ns gating
- 500 kHz repetition rate

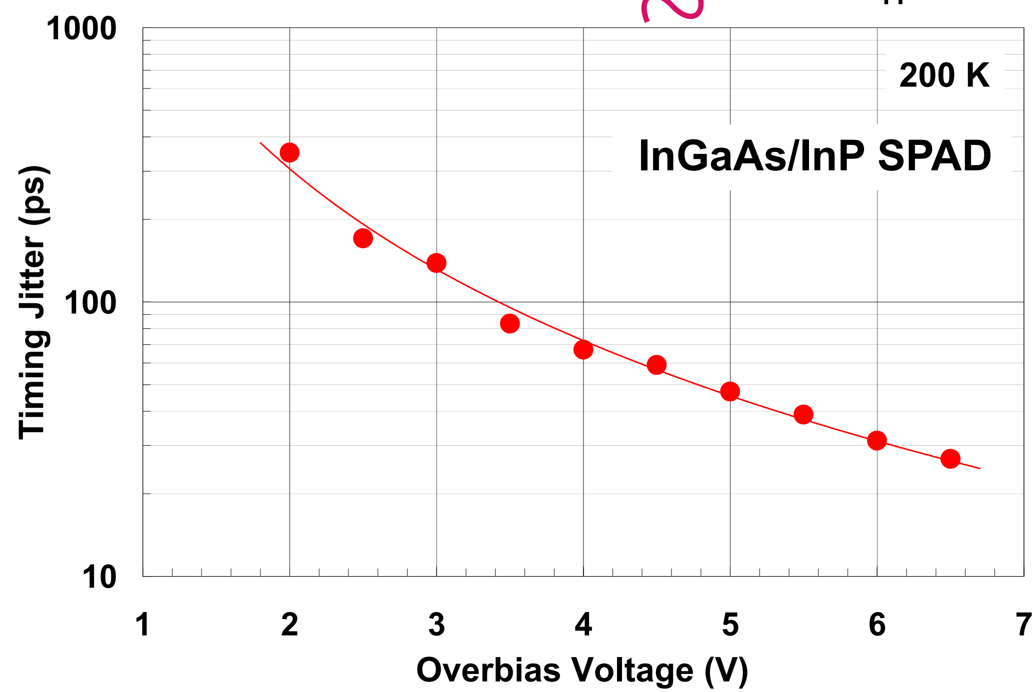
# Timing Jitter

- **Several factors affect detection timing**
- **Can be on par with other fast SPC detectors**
  - Silicon SPADs ~ 50 ps
  - Superconducting SPDs ~ 30 ps
  - Requires high excess bias
    → DCR and afterpulsing trade-offs
- **Jitter often circuit-limited**




*Zappa, Tosi, Cova, SPIE 65830E (2007)*




*Itzler, et al., J. Modern Opt. 54, 283 (2007)*

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# Challenges of high-rate photon counting

**Two essential challenges for high-rate counting with APDs:**

- **Suppress high-frequency transients**
  - Transients are common artifact of high bandwidth signal modulation
- **Suppress afterpulsing**
  - Afterpulsing elevates dark counts due to carrier trapping/detrapping
  - Historical mitigation by long hold-off times not an option at high rates

Must also have sufficient intrinsic device bandwidth

→ not a problem for small-diameter InP/InGaAs APDs up to ~GHz-scale

# Transients induced by fast signal modulation


fast rise of gate voltage

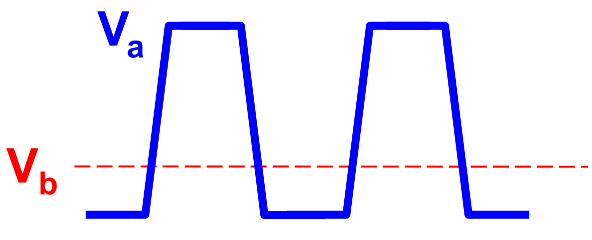
transient response

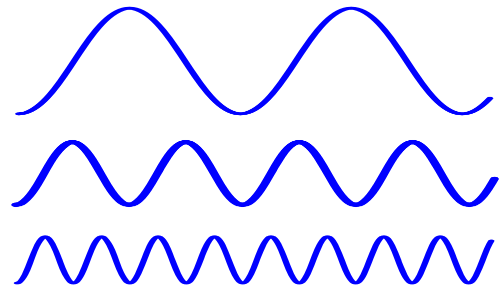
**Dominant strategy: create identical transient and subtract**

- **Obtain matched transient from another circuit element**
  - Matched APD detectors (NEC)
  - Dummy capacitance of appropriate size (Politecnico di Milano)
- **Obtain matched transient from same circuit element**
  - Matched RF delay lines (IBM → PLI)
  - Self-differencing by periodic delay and subtract (Toshiba/UK)

# Consider range of modulation techniques

- **"High-slew" gating → canonical approach**

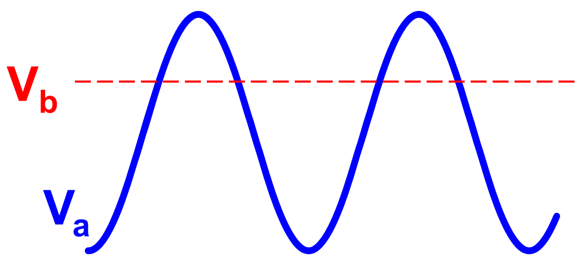


harmonics give rise to transient "ringing"

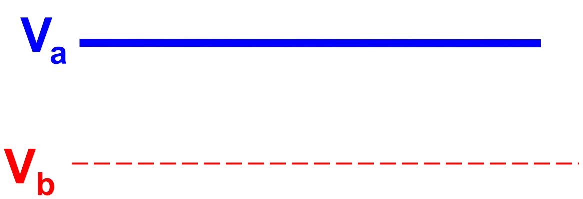
- **Single-frequency gating → "sine-wave" gating approach**

Nihon U., Geneva U.




No harmonics!
Just filter at the gating frequency

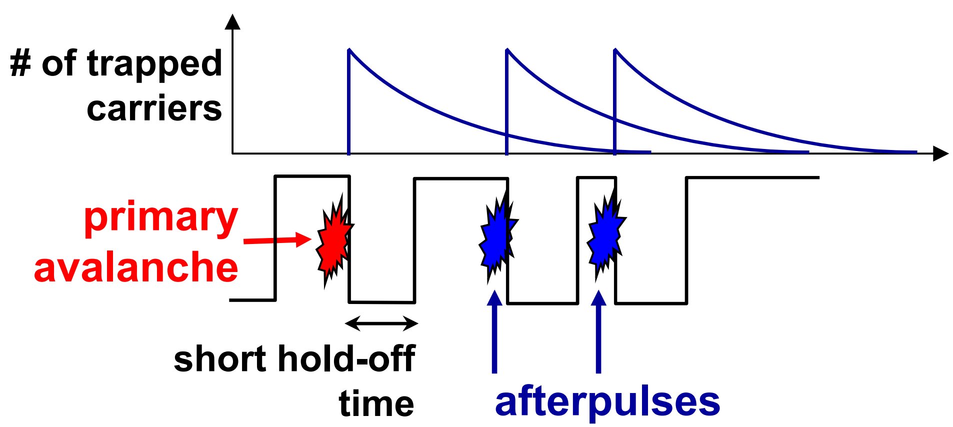
- **"DC" biasing → "self-quenching" approach**

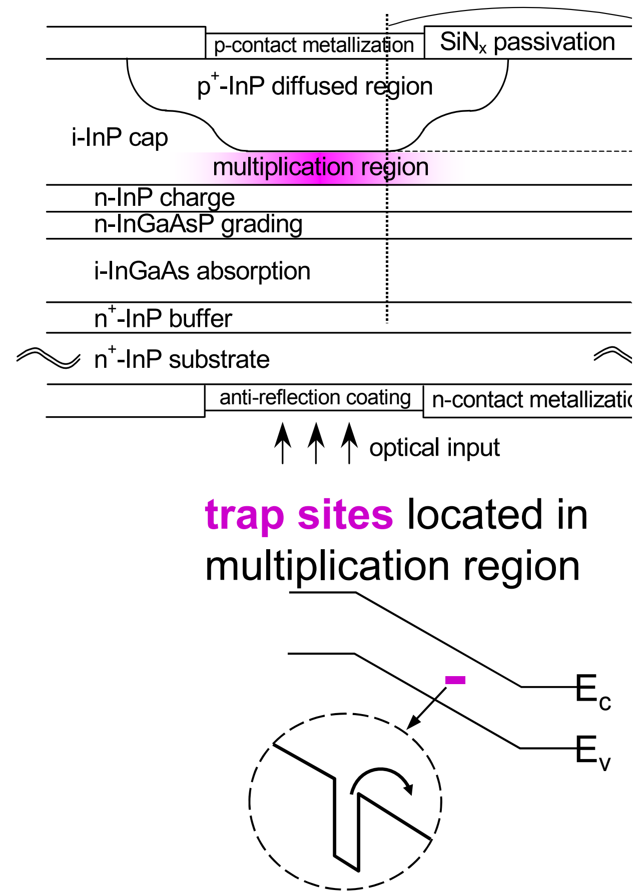



No transients!
Any signal is due to avalanche event

# Afterpulsing: increased DCR at high rate

- **Single photon detection by avalanche multiplication in SPADs**
- **Avalanche carriers trapped at defects in InP multiplication region**
- **Carrier de-trapping at later times initiates "afterpulse" avalanches**



# Afterpulsing:  increased DCR at high rate



- **Single photon detection by avalanche multiplication in SPADs**
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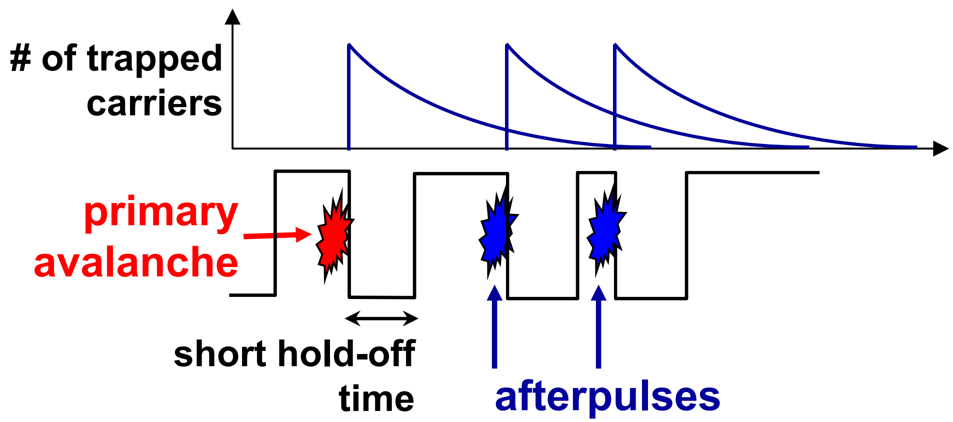
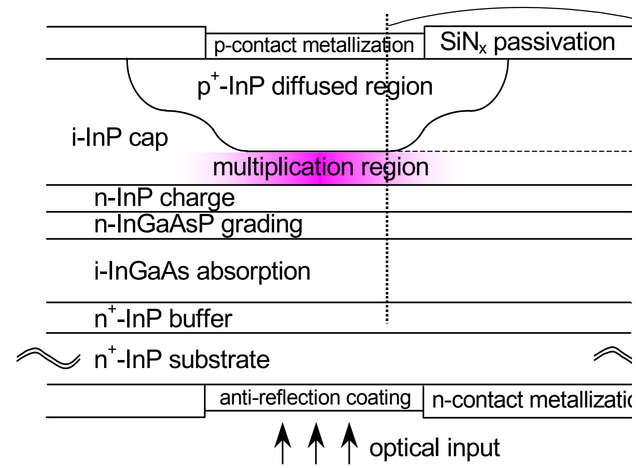
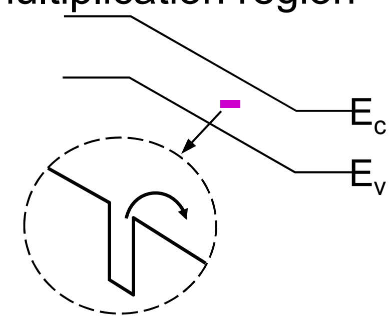
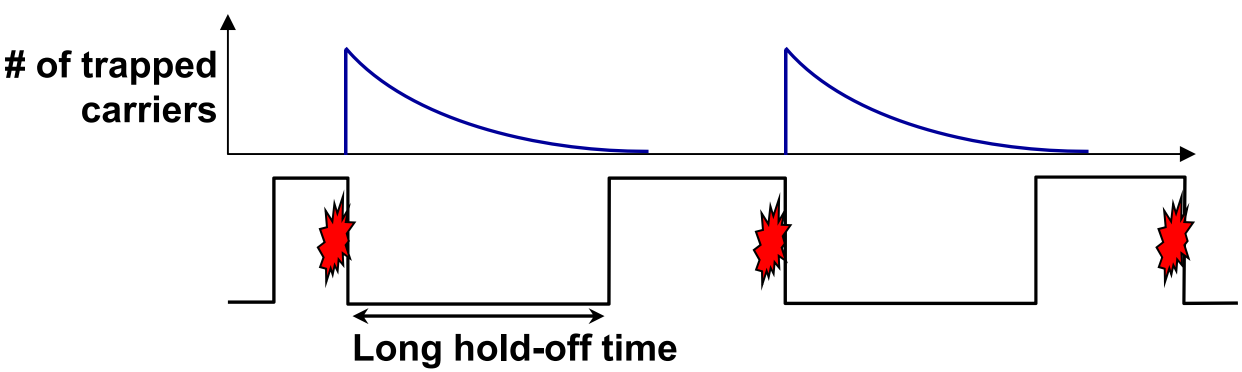
- **Serious drawback of afterpulsing → limitation on counting rate**




trap sites located in multiplication region

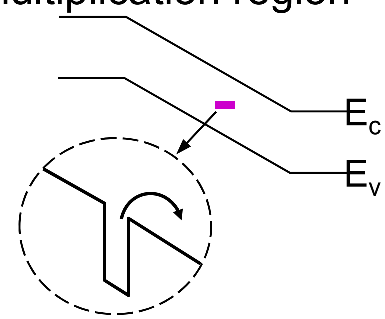


# Afterpulsing suppression strategies

- **Sufficient hold-off time before re-arming**
  → low repetition rate
- **Reduce material defects that cause trapping**
  → defects not known; substantial materials challenges
- **Rapid intentional detrapping by applied stimulus**
  → optical stimuli (sub-bandgap) not successful to date
  → thermal stimuli involve thermal time constants, probably too slow
- **Reduce number of trapped carriers**
  → **reduce charge flow per avalanche**
  - requires some form of rapid quenching → **strong "negative feedback"**
  - **consistent with high-speed gating** (short gates reduce charge flow)

**trap sites** located in multiplication region

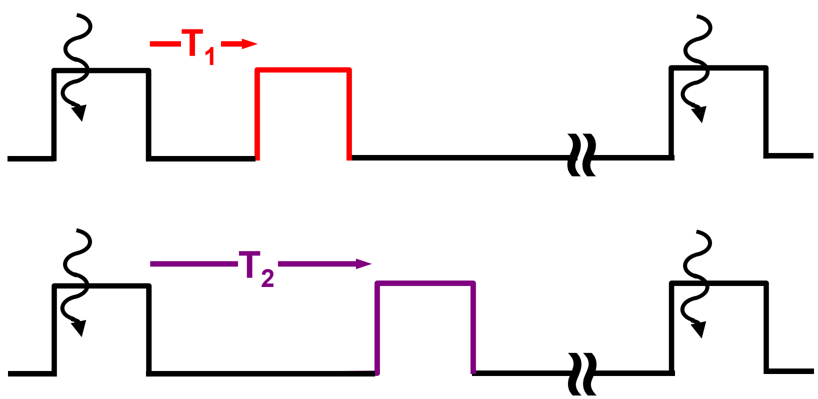


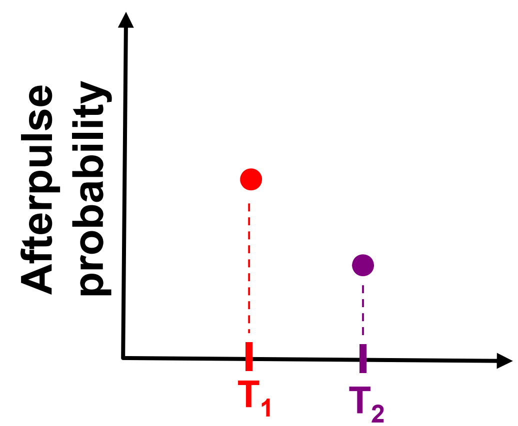
# "Double-pulse" afterpulse characterization

- Use "time-correlated carrier counting" technique to measure afterpulses
- Trigger single-photon avalanches in 1st gate
- Measure probability of afterpulse in 2nd gate at $T_n$
- Use range of $T_n$ to determine dependence of afterpulse probability on time following primary avalanche

Cova, Lacaita, Ripamonti,
EDL **12**, 685 (1991)

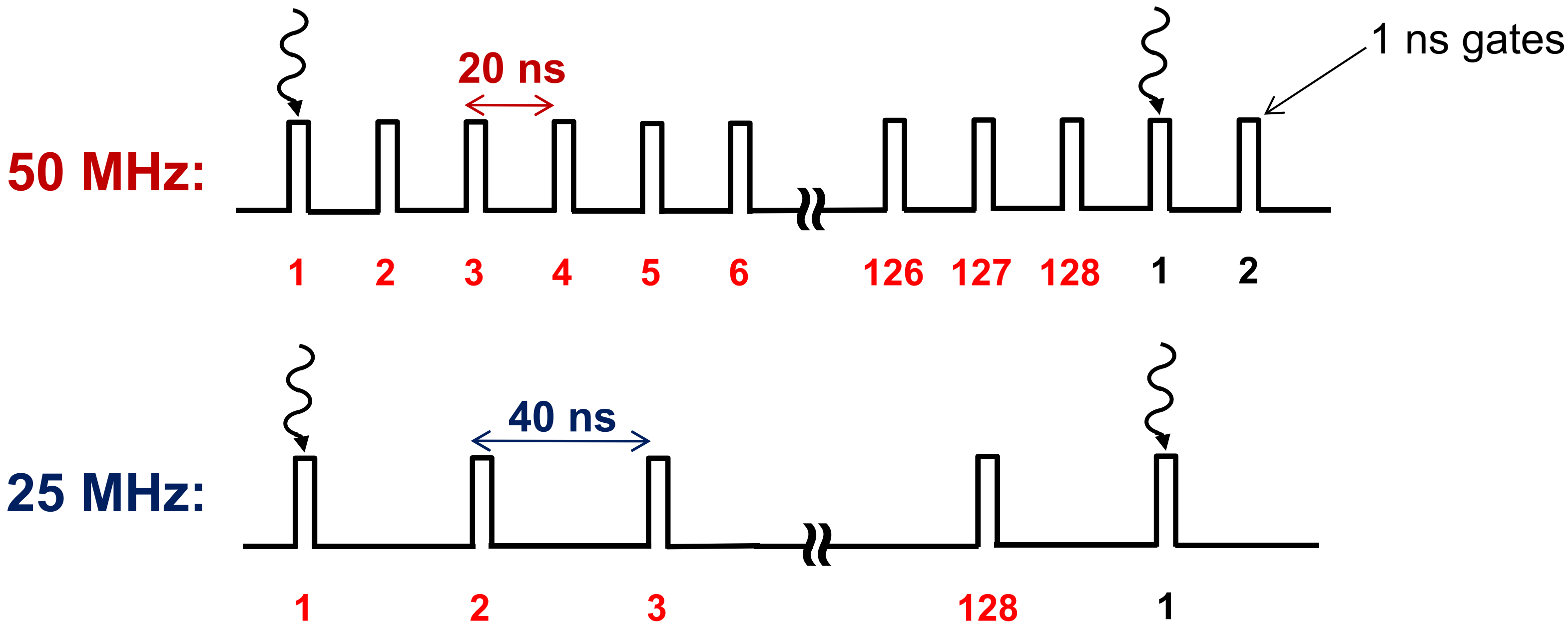
*Double-pulse ("pump-probe") method*



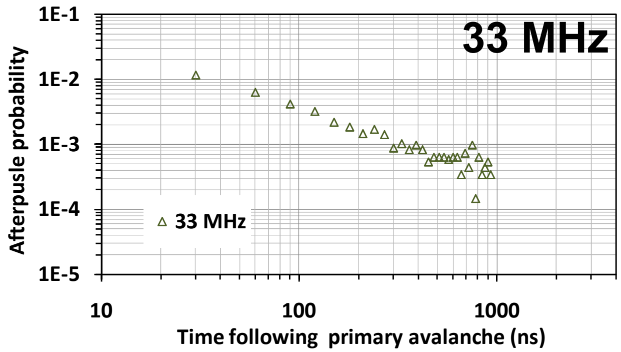
# FPGA-based data acquisition

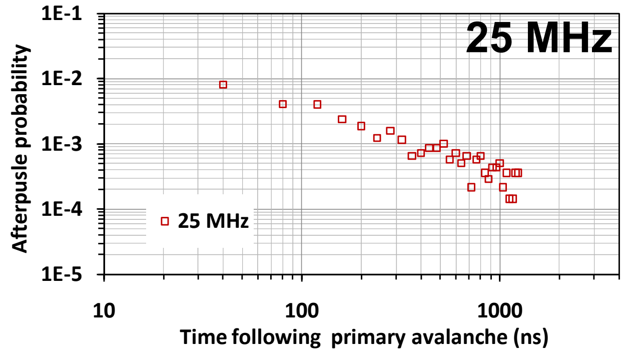
- **Use FPGA circuitry to control gating and data collection**
- **Generalize double-pulse method to many gates**
  - Capture afterpulse counts in up to 128 gates following primary avalanche
  - Temporal spacing of gates determined by gate repetition rate
- **Allows capture of afterpulse count in any gate after avalance**
  - No need to step gate position as in double-pulse method

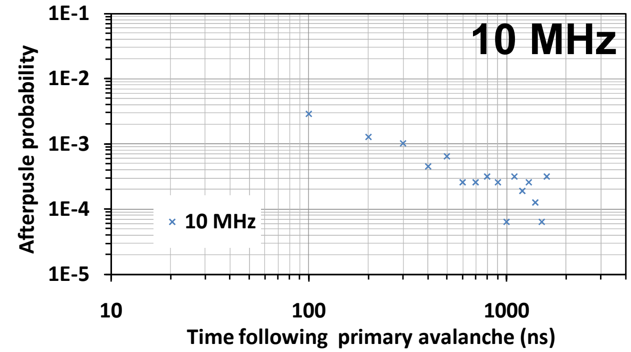


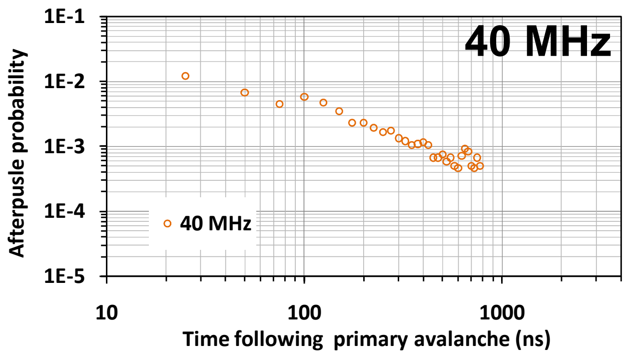
# FPGA-based afterpulse measurements

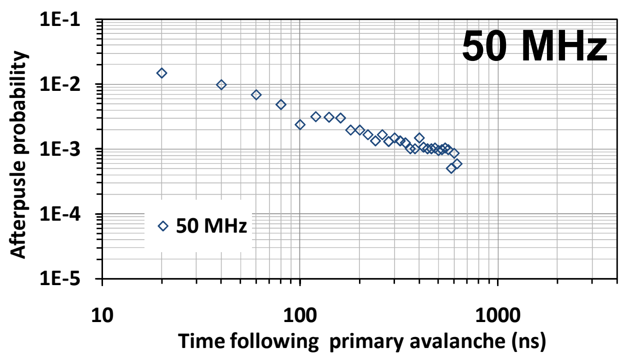
- **Obtain afterpulsing probability data at 5 frequencies for 32 gates**

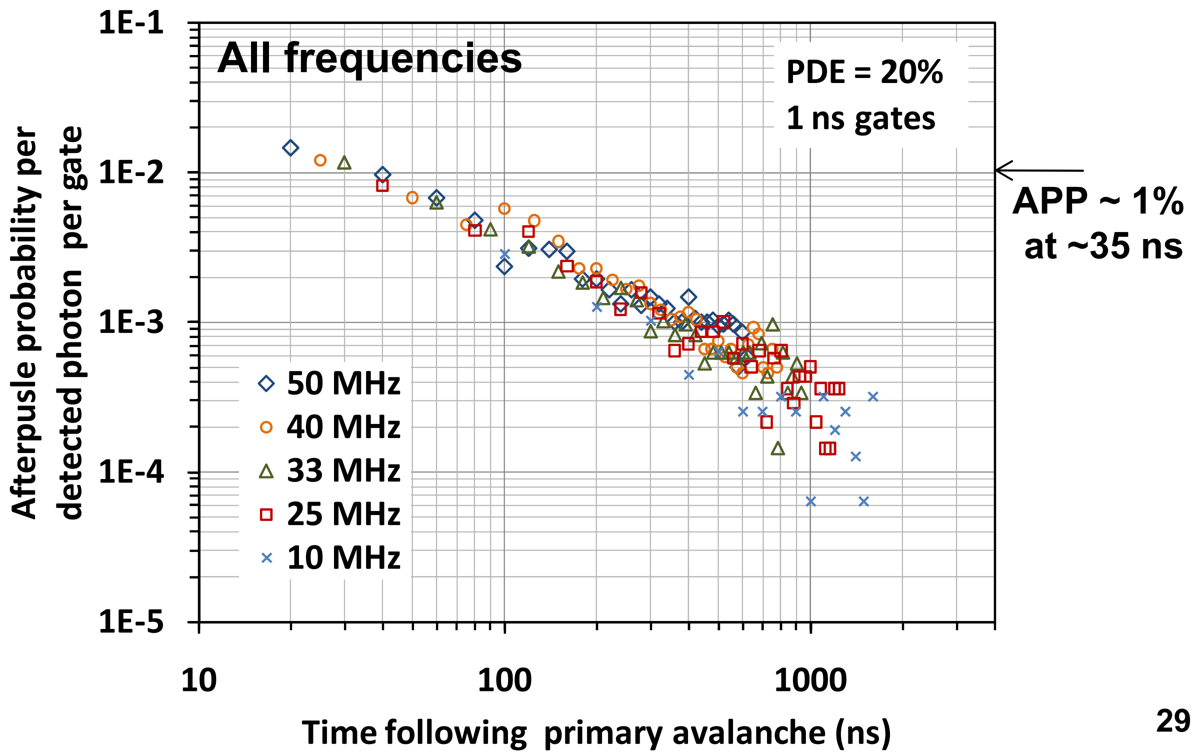


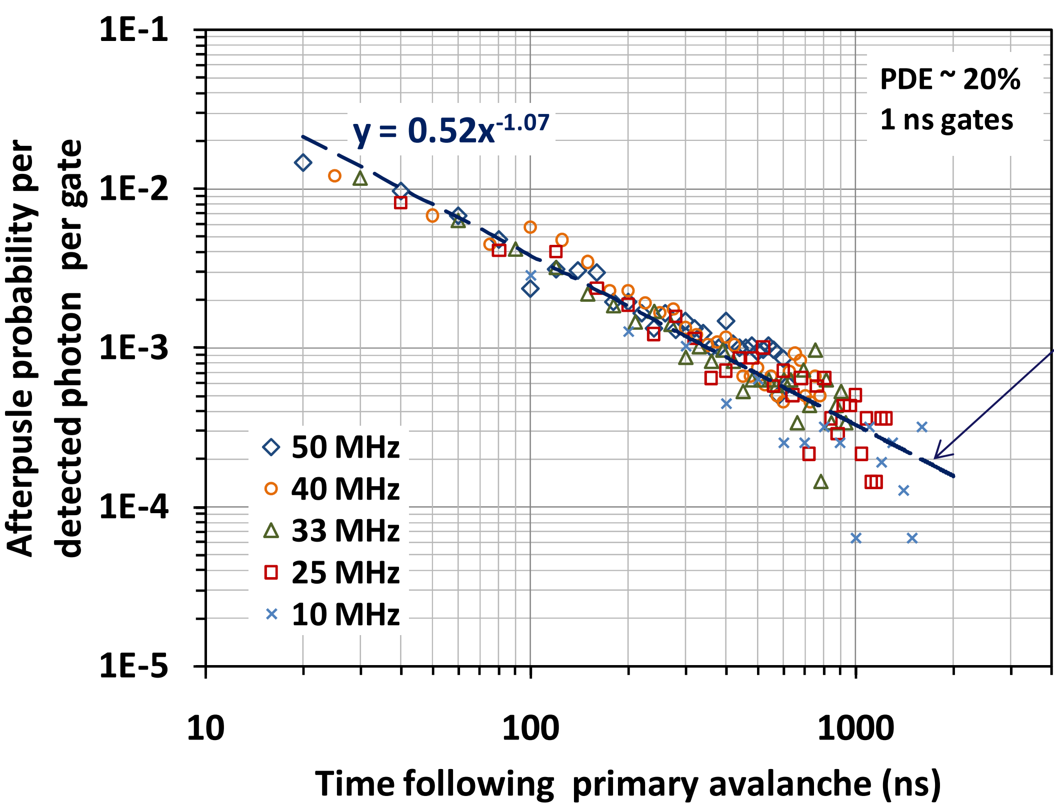



# Recent re-interpretation of afterpulsing behavior

- **Past fitting has assumed exponentials but is completely arbitrary**
- **We found good fitting for simple power law $T^{-\alpha}$ with $\alpha \approx -1$**

  **→ Is power law behavior found for other afterpulsing measurements?**

  **→ Is the power law functional form physically significant?**



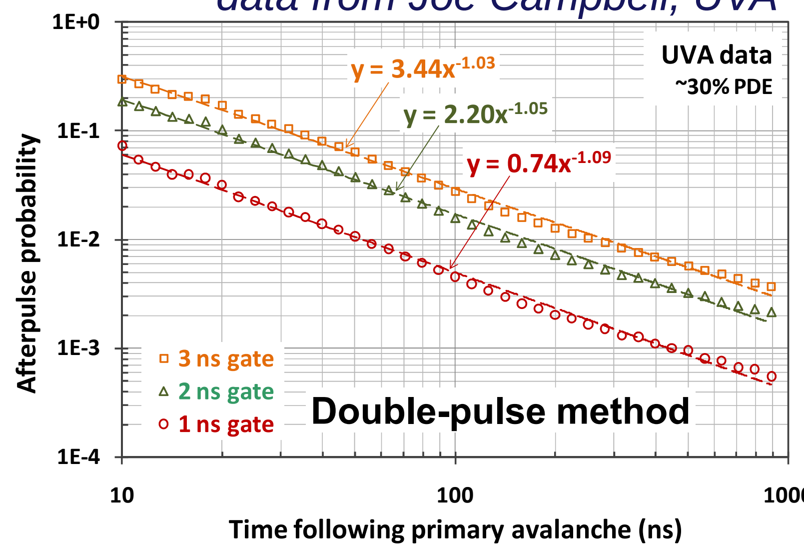

**APP = C $T^{-\alpha}$**

Princeton Lightwave

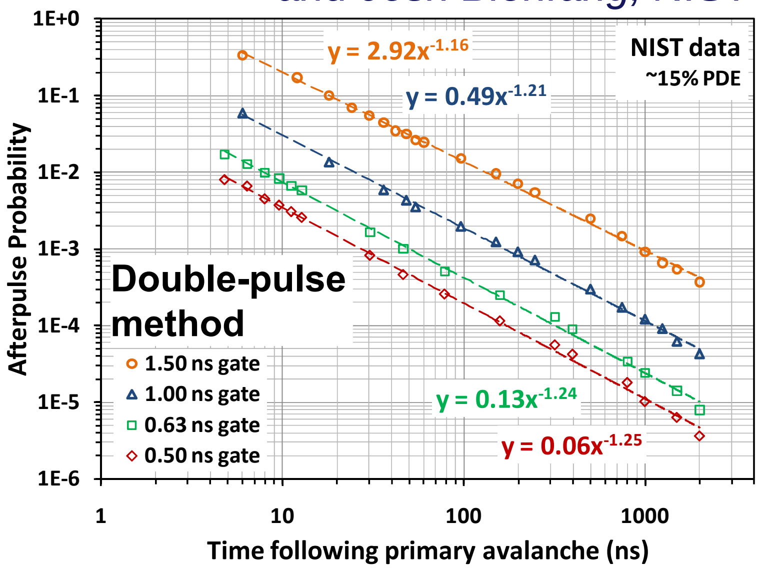
# Afterpulsing data from other groups

- **Good fits for power law** $T^{-\alpha}$ **with** $\alpha \approx -1.0$ **to -1.4**
- All data for PLI InGaAsP SPADs

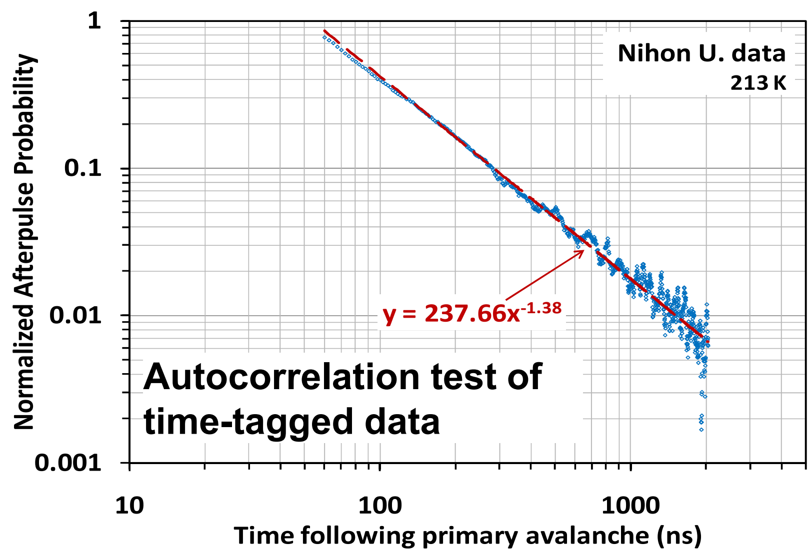
*data from Joe Campbell, UVA*




*data from Alessandro Restelli and Josh Bienfang, NIST*



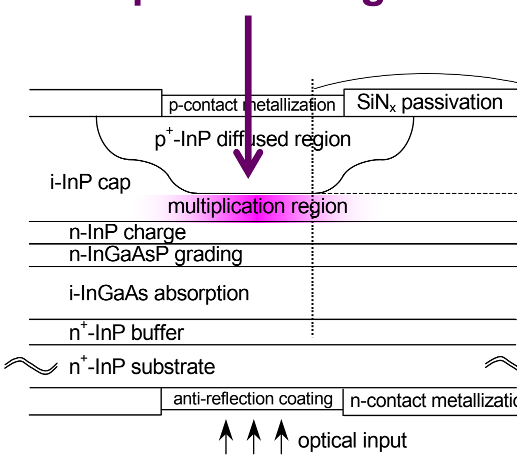

*data from Naota Namekata, Nihon U.*

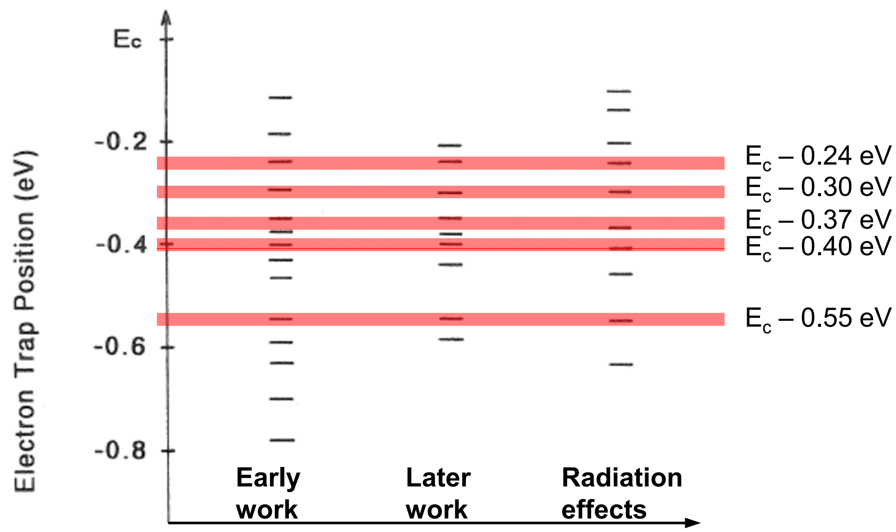


# Literature on InP trap defects

- **Literature on defects in InP describes dense spectrum of levels**
- **Power law behavior consistent with distribution of detrapping times**
  - Based on simple model of afterpulsing with distribution of defects
  - Accurate only for specific distribution with $D(\tau) \propto \tau$

**Deep-level traps in multiplication region**

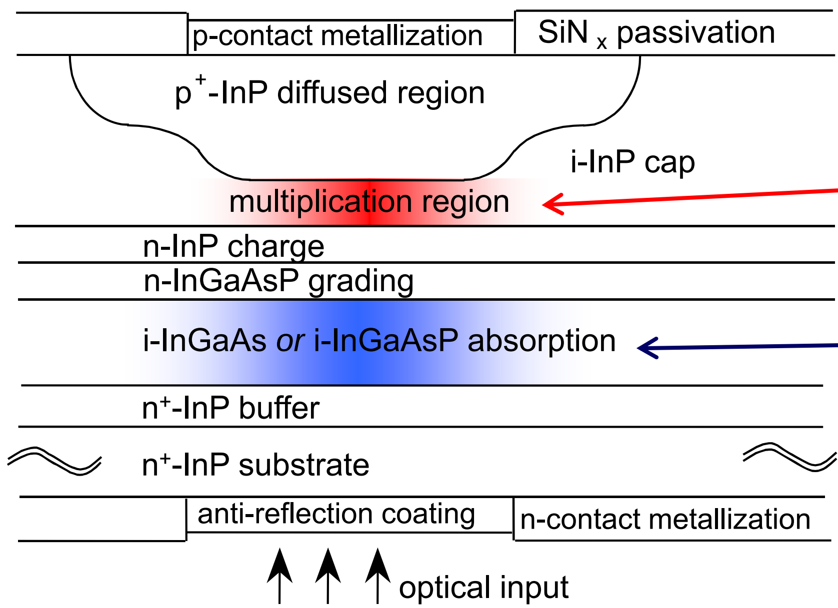



**W. A. Anderson and K. L. Jiao, in "Indium Phosphide and Related Materials: Processing, Technology, and Devices", A. Katz (ed.) (Artech House, Boston, 1992)**

# Dark counts: possible connection to afterpulsing?

- **Dark counts dominated by two mechanisms in SPAD structure**




**Trap-assisted tunneling in large-bandgap (~1.35 eV) InP multiplier**

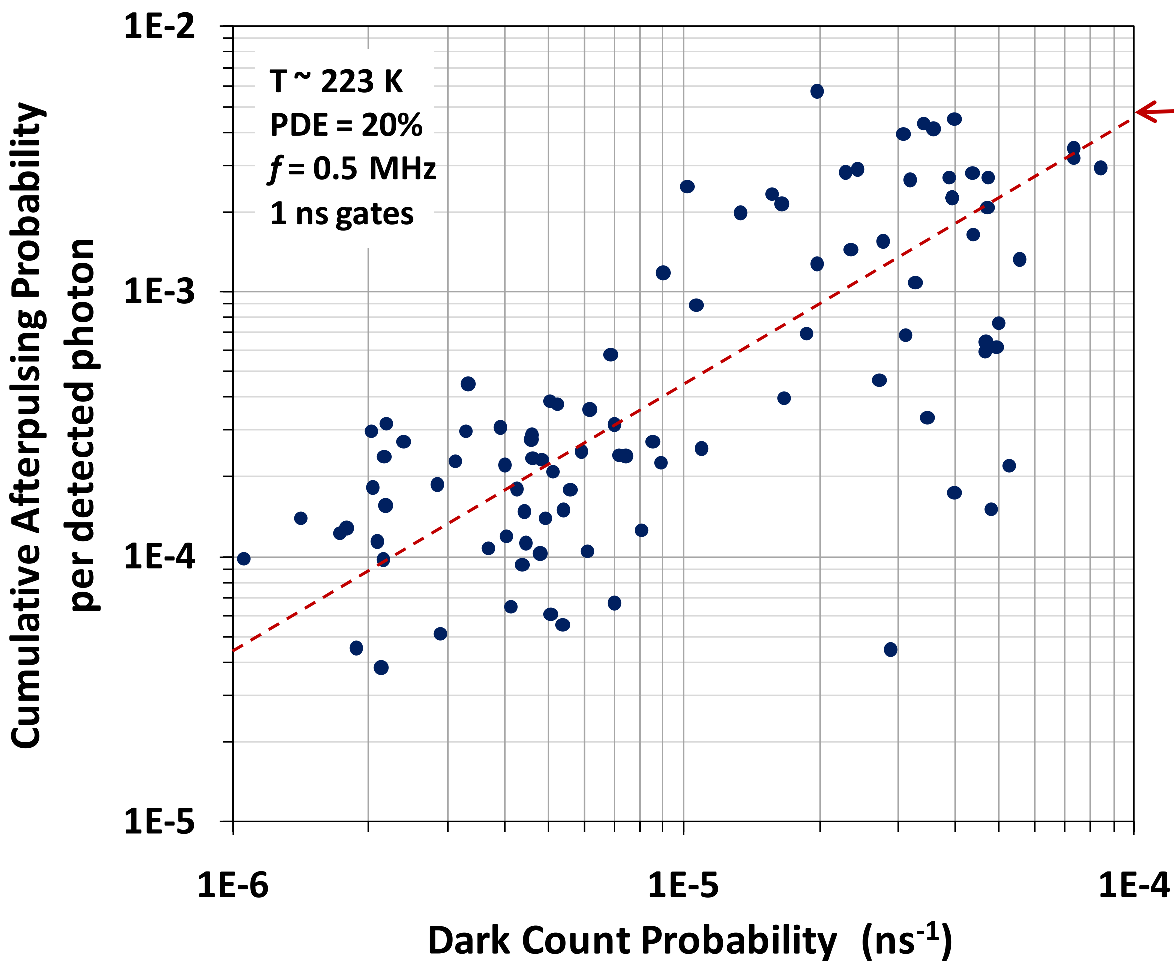
**Thermal generation in small-bandgap (~0.77 ev) InGaAs absorber**

- **Afterpulsing is caused by carrier trapping in multiplier**

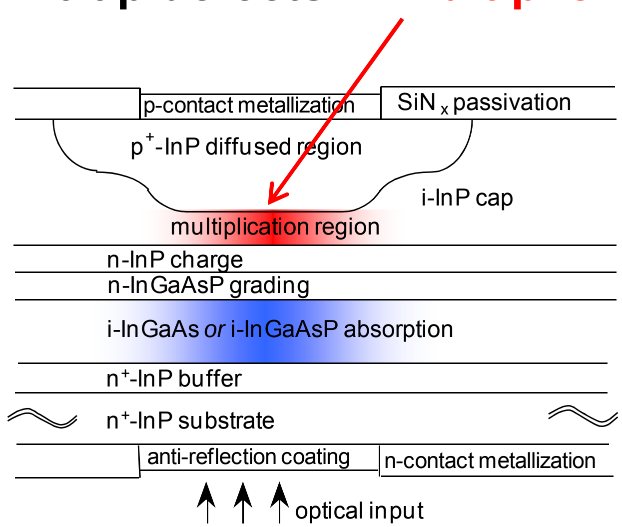
**→ Are TAT-induced dark counts and afterpulses due to same traps?**

# Correlation of afterpulsing with DCR

- **First evidence for same traps causing TAT and afterpulsing**
  - Scatter is large, so large sample size (~100 devices) is required
  - **To have low afterpulsing, must have low TAT-induced DCP**

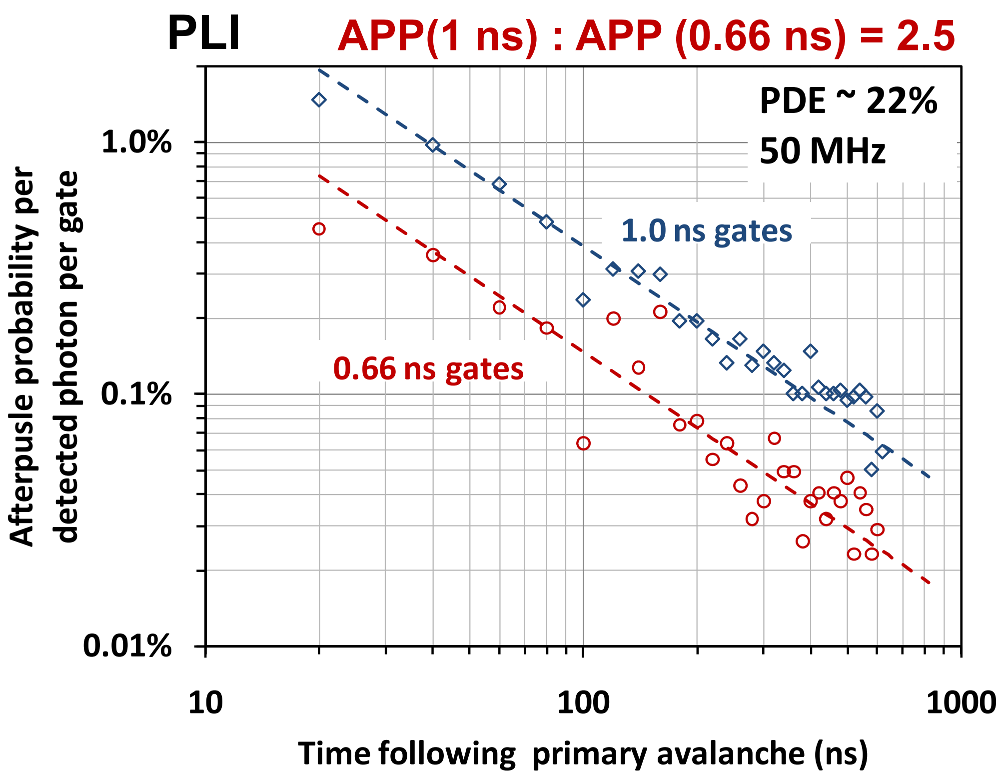


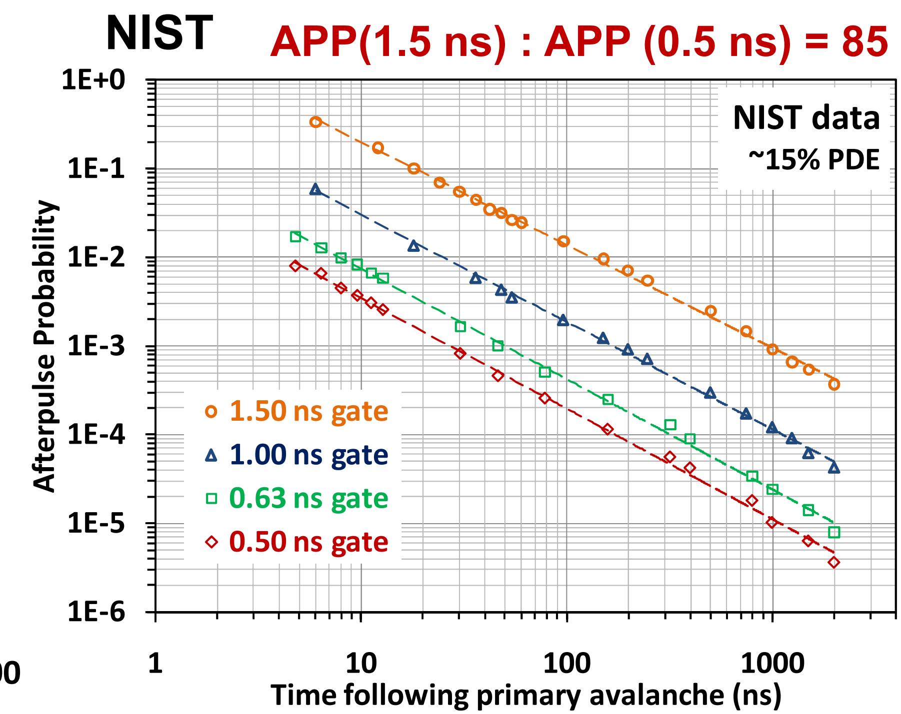

**trap defects in multiplier**



# Afterpulsing reduction with shorter gates

- **Two advantages inherent in using shorter gates**
  - **Shorter "window"** in which afterpulse can be detected → linear in gate width
  - **Charge flow reduction** → net reduction in APP is super-linear in gate width
- **Enables higher counting rates with "synchronous counting"**




*Data from A. Restelli and J. Bienfang*

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- InGaAsP SPAD fundamentals
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- **High-rate photon counting with InGaAsP SPADs**
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  - Integration of negative feedback
  - Self-quenching avalanche dynamics
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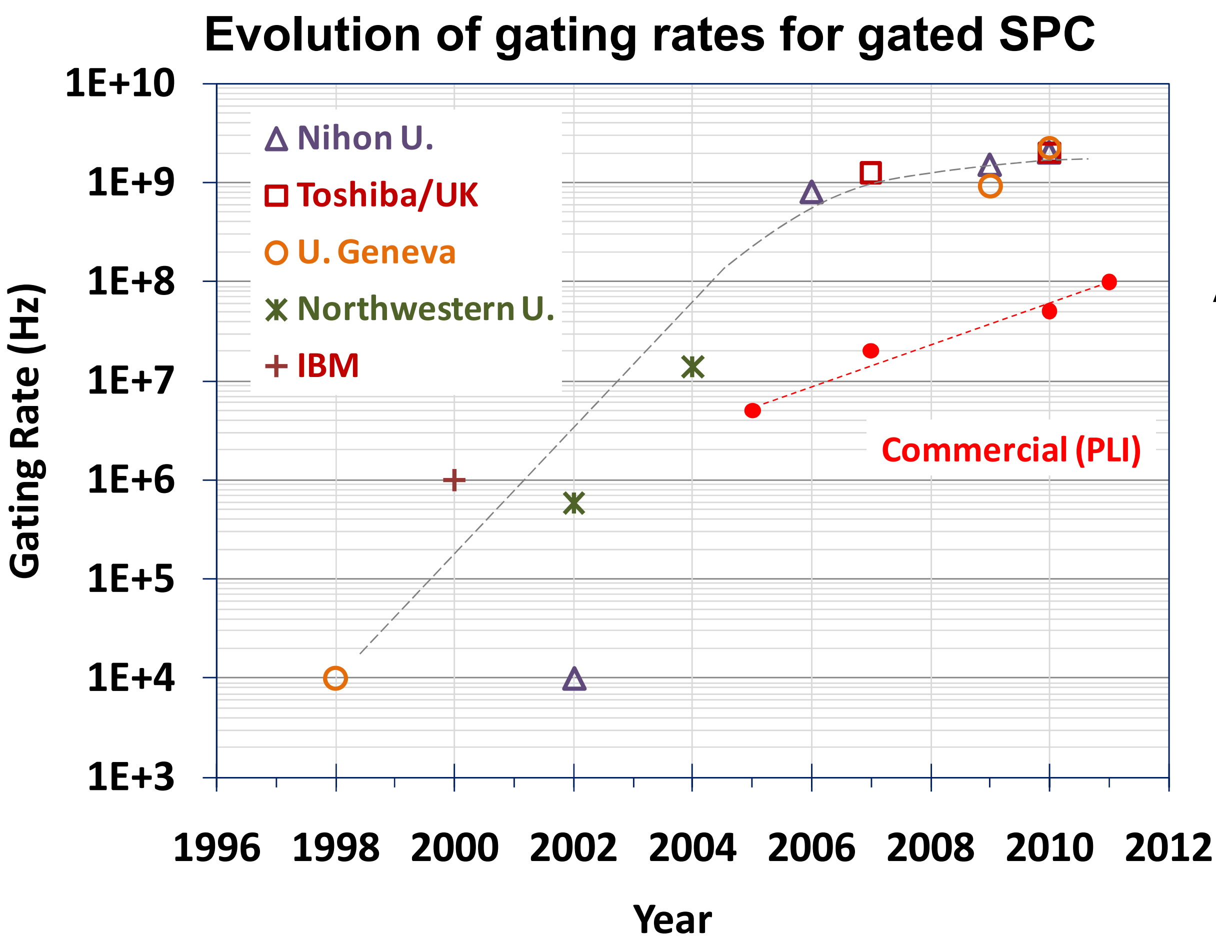
# Evolution of photon counting rate

- **Higher counting rate → shorter gates → reduced afterpulsing**
- **Gating rate is most consistent metric with sufficient data**
  - Several rate metrics: System clock, periodic gating rate, actual counting rate




**PDE ~ 10 – 20%**

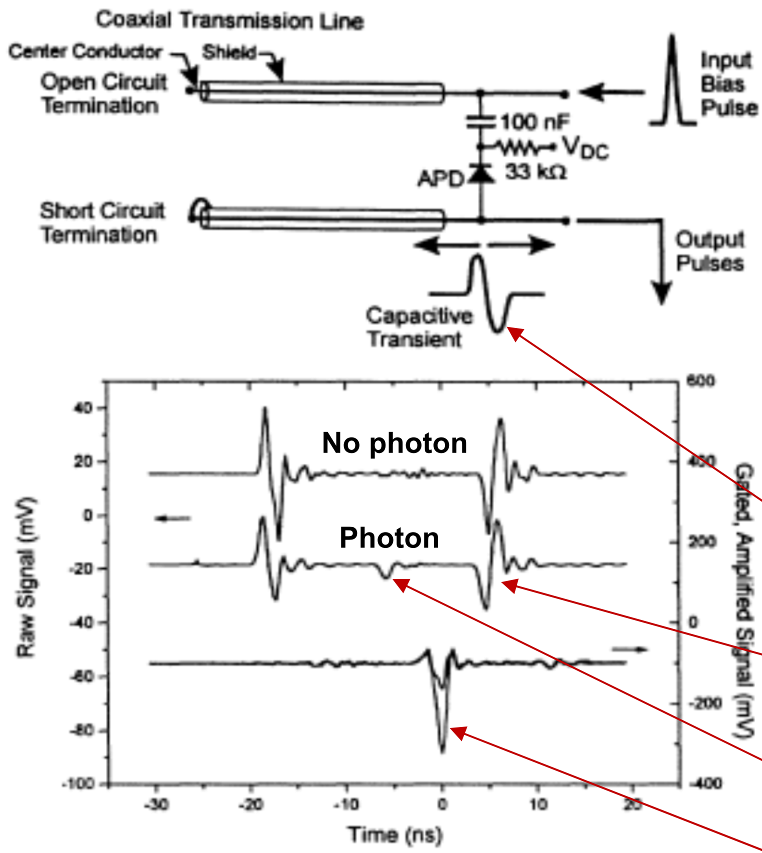
**Acceptable afterpulsing (APP ≲ 5%)**

M. A. Itzler

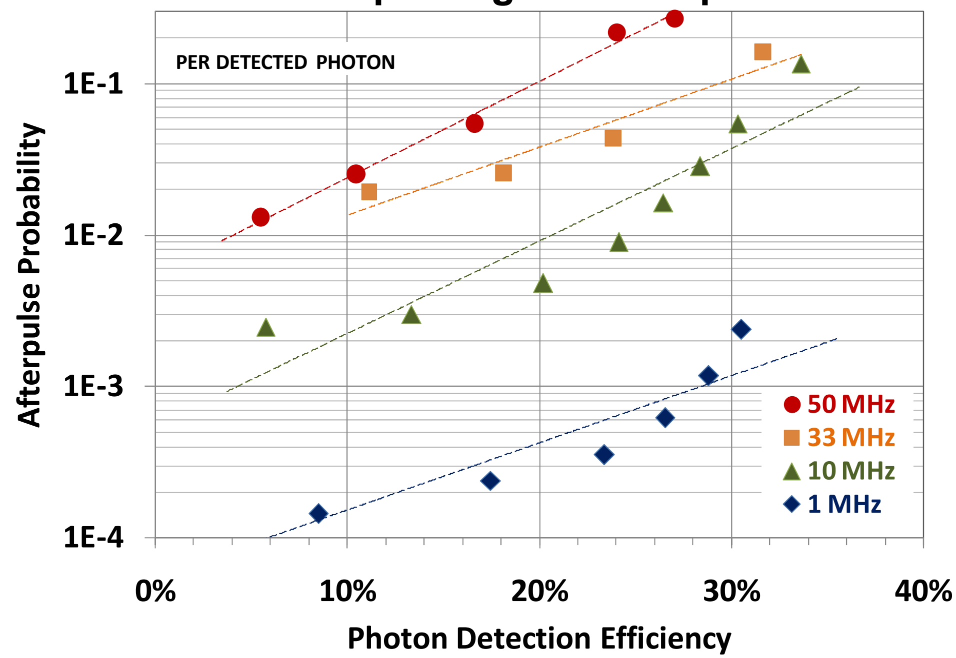
# Transient cancellation with RF delay lines

- **Precise cancellation for reduced threshold → detect smaller avalanches**
- **Afterpulsing ~ 3% at 12% PDE at 50 MHz**
  - **100 MHz now available commercially**

***D. Bethune and W. Risk, JQE 36, 340 (2000)***




**Afterpulsing vs PDE up to 50 MHz**




Cancel transient response coincident with photon arrival

Temporally gate out leading and trailing transients

Net signal

Signal after amplification and temporal gating

# Self-differencing up to 2 GHz

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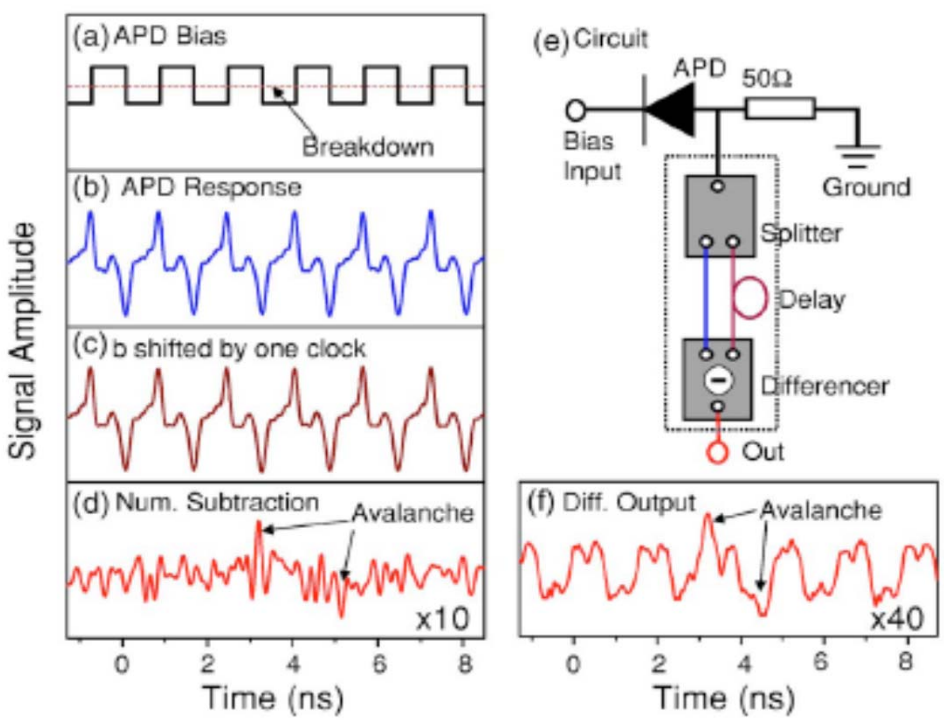
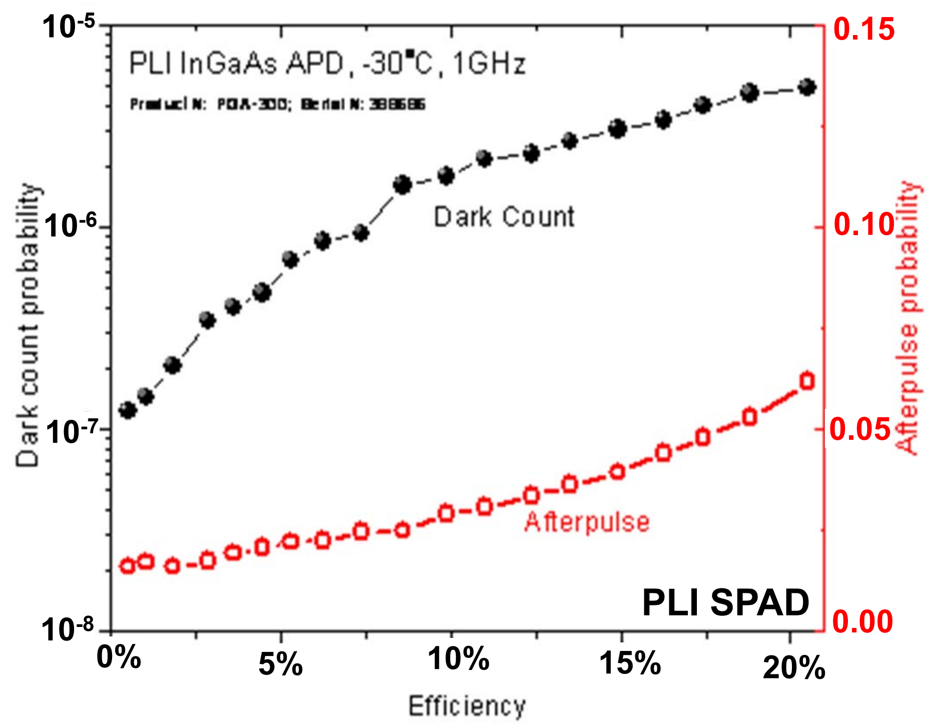
- **Toshiba self-differencing technique with GHz gating, sub-ns gates**
  - 2 GHz gate repetition frequency, 50% duty cycle
- **Afterpulsing ~1.5% (at 12% PDE) demonstrated at 2 GHz**

***Yuan, et al., APL 96, 071101 (2010)***



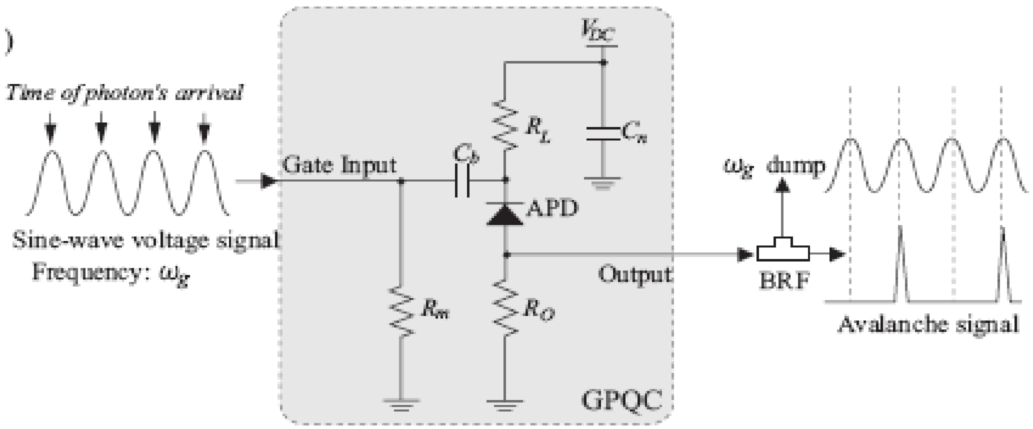

***Z. Yuan, et al., Appl. Phys. Lett. 91, 041114 (2007)***

***courtesy of Zhiliang Yuan – Toshiba/UK***

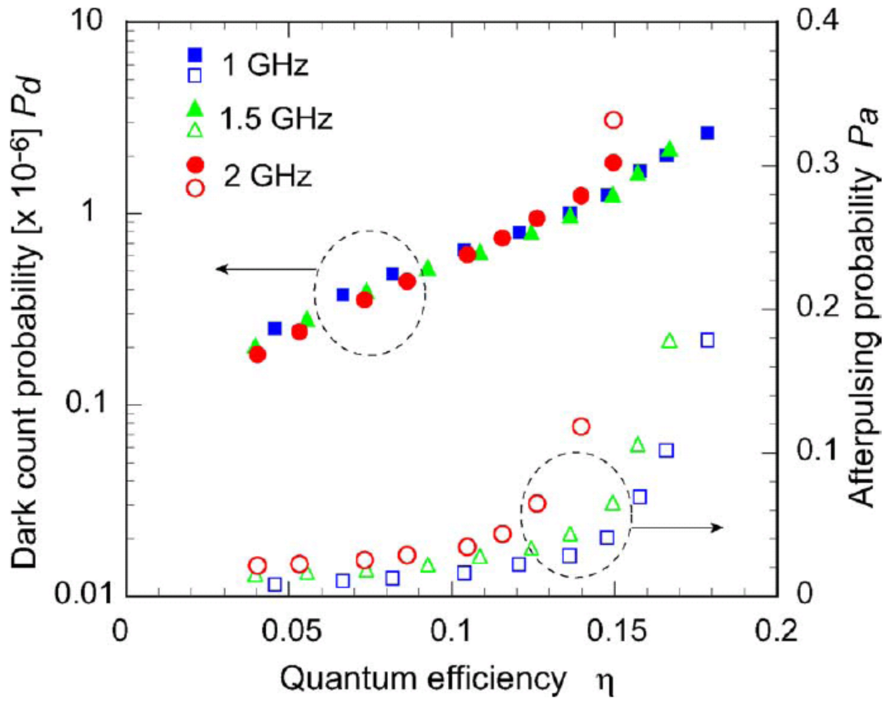
# Sub-ns gating at 2 GHz with sine wave gating

- **Nihon Univ. sine-wave gating up to 2 GHz, sub-ns gates**
  - Strong notch filtering of sine wave bias leaves only avalanche response
- **Afterpulsing probability ~5 % (at 12% PDE) at 2 GHz**

**Sine wave gating and filtering**




***N. Namekata, S. Sasamori, S. Inoue, Opt. Expr. 14, 10043 (2007)***




***N. Namekata, S. Sasamori, S. Inoue, PTL 22, 529 (2010)***

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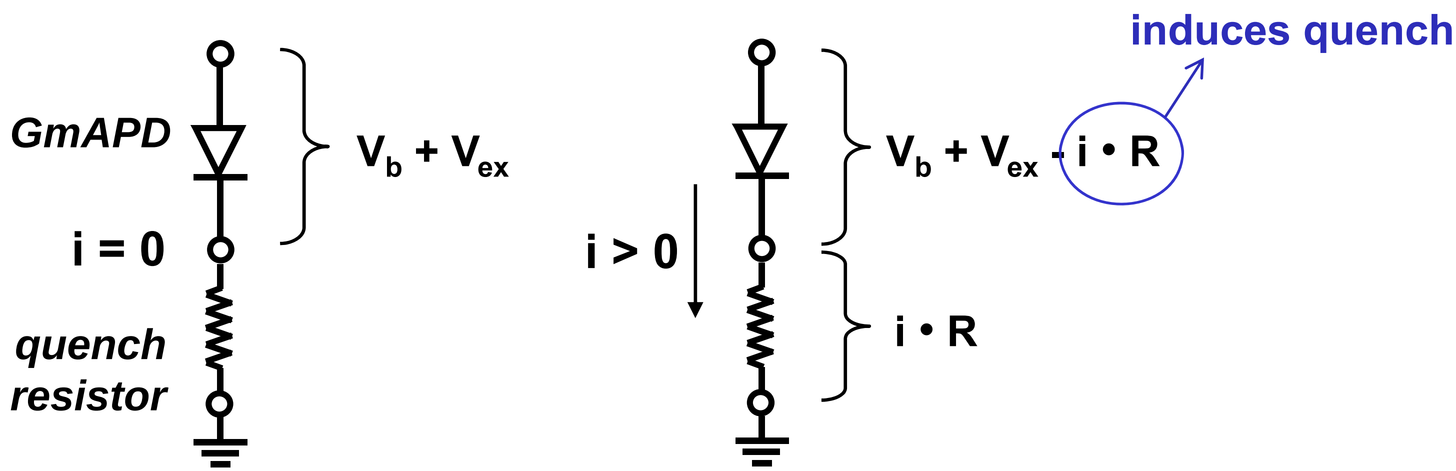
# Self-quenching "negative feedback" APD (NFAD)

- **Can we mitigate afterpulsing and crosstalk w/o complexity of short gating?**
- **Reduce avalanche current flow by self-quenching**

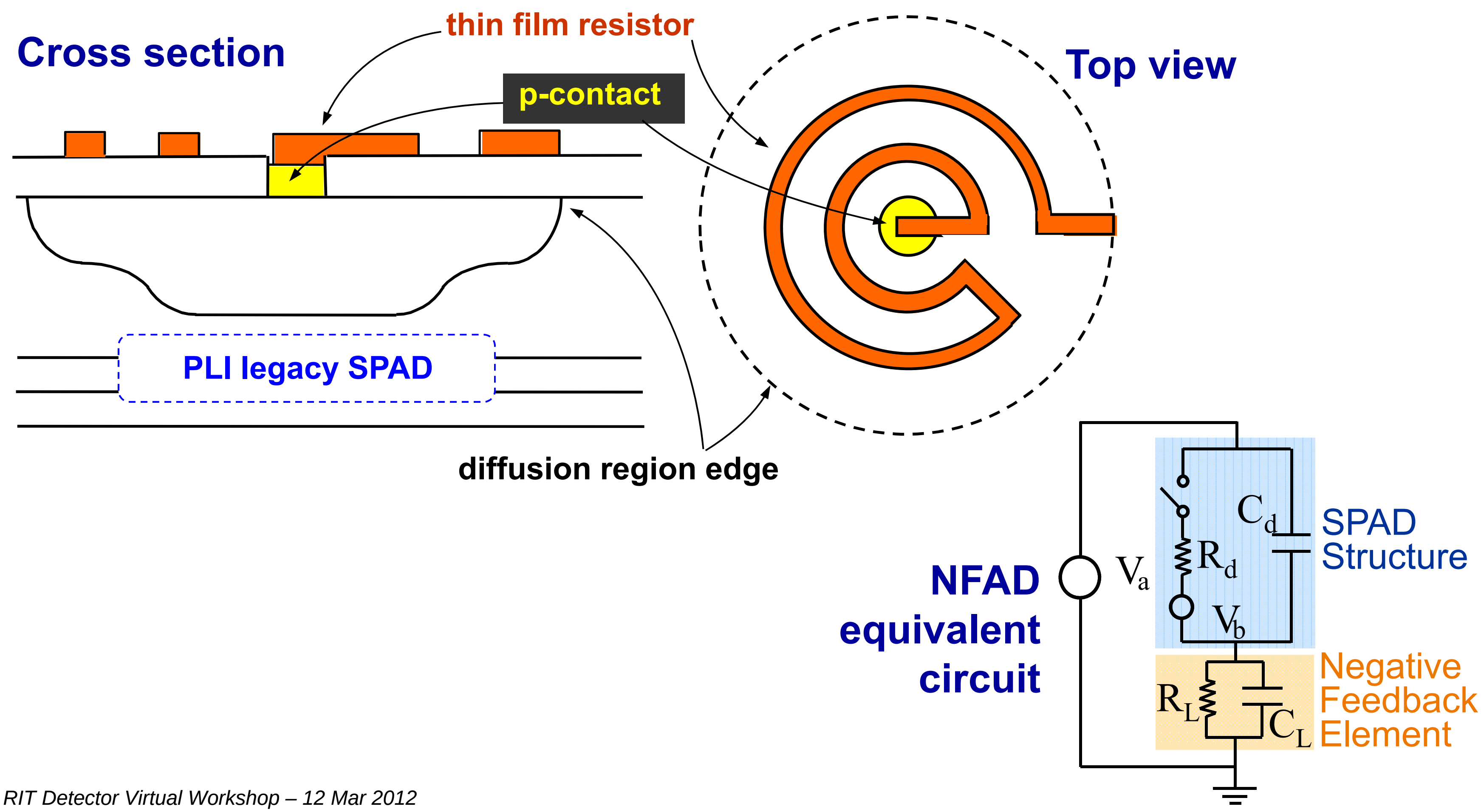
  **→ Introduce "negative feedback" to oppose the positive feedback of avalanche impact ionization process**

- **Use passive quenching with "free-running" detector**
  - Fixed DC bias across GmAPD + Resistor
  - Current flow through load resistance causes I • R drop → shifts voltage away from SPAD



# Self-quenching NFADs device design

- **Use monolithic implementation to minimize parasitic effects**
  - Surface-integrated thin film resistors
  - Fully compatible with optimal GmAPD designs – no epi-structure tradeoffs



# Self-quenching behavior depends on feedback

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- **Need large $R_L$ to ensure rapid self-quenching and small charge flow Q**
  - Current in junction must fall below threshold value for self-quench to occur
- **"Recharge" time following quench has time constant $R_L C_d$**

**Principal design trade-off:**
**Large $R_L$ → rapid quenching**
**Small $R_L$ → rapid recharging**

***Device diameter:*** **25 μm**

***Discharge (quench)*:**

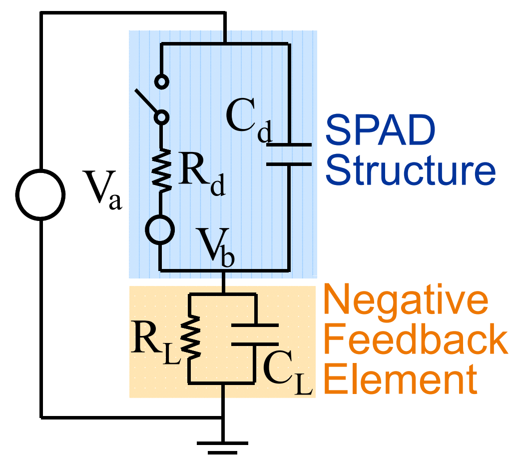
$\tau \sim R_d C_d \rightarrow (5\ \mathrm{k\Omega})(100\ \mathrm{fF}) \sim 0.5\ \mathrm{ns}$

***Recharge (re-arm)*:**

$\tau \sim R_L C_d \rightarrow (0.1 - 1\ \mathrm{M\Omega})(100\ \mathrm{fF}) \sim 10 - 100\ \mathrm{ns}$

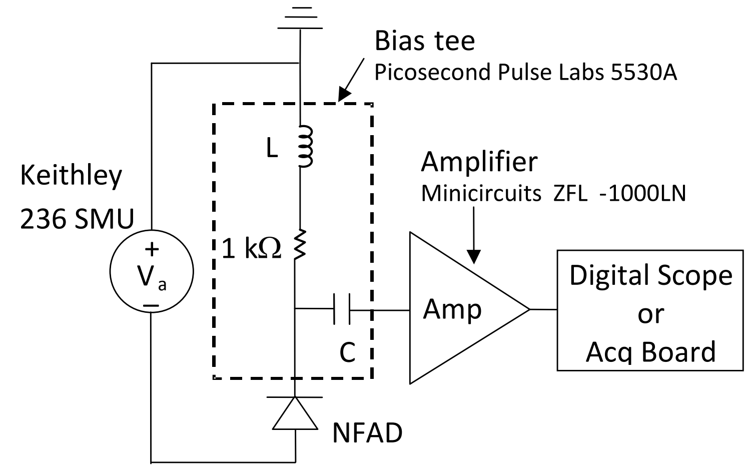
***"Minimum" charge flow*:**

$Q = C_d V_{ex} \rightarrow (100\ \mathrm{fF})(2\ \mathrm{V}) \sim 1 \times 10^6\ e^-$

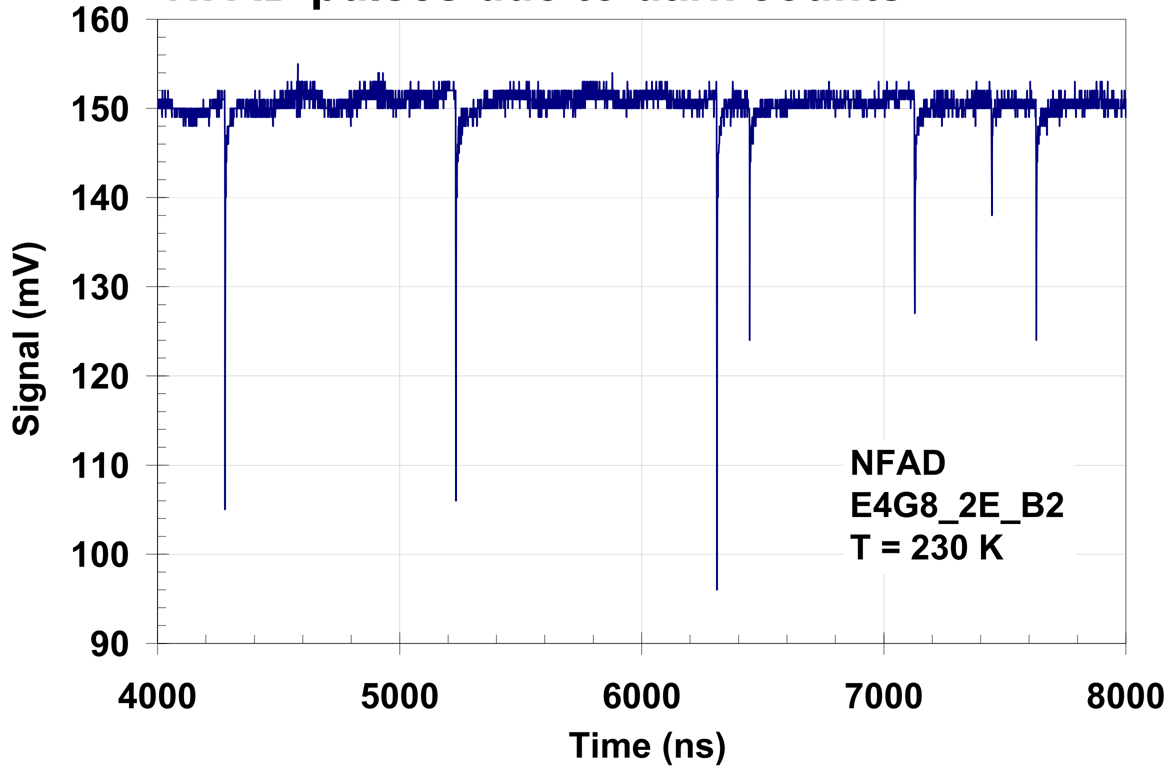


# First generation of NFAD devices exhibited desired behavior

- Operate with simple bias T
- Sharp detection pulses with fixed DC bias




## NFAD pulses due to dark counts




$R_L$ ~ 90 kΩ

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- Operate with simple bias T
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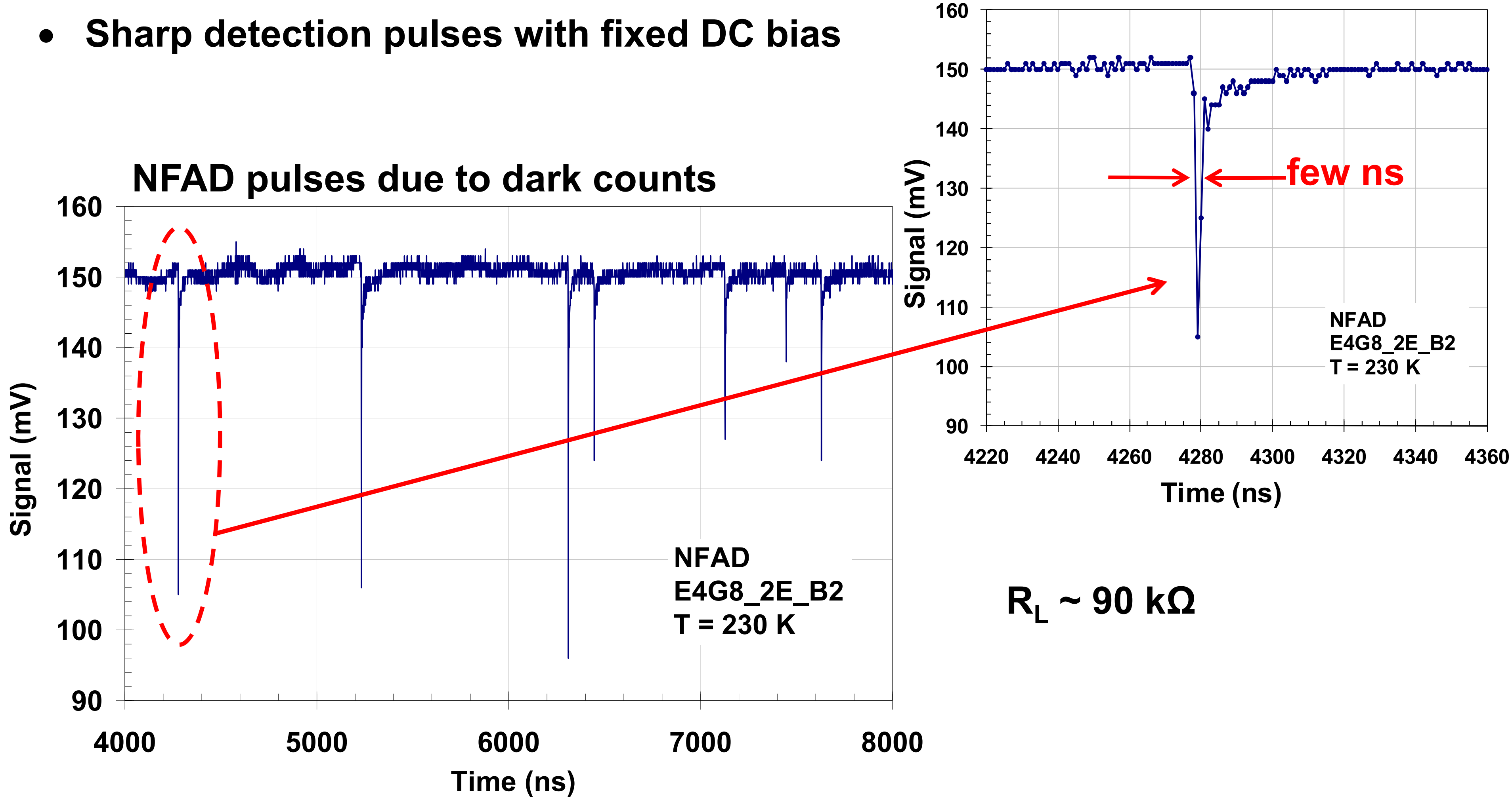


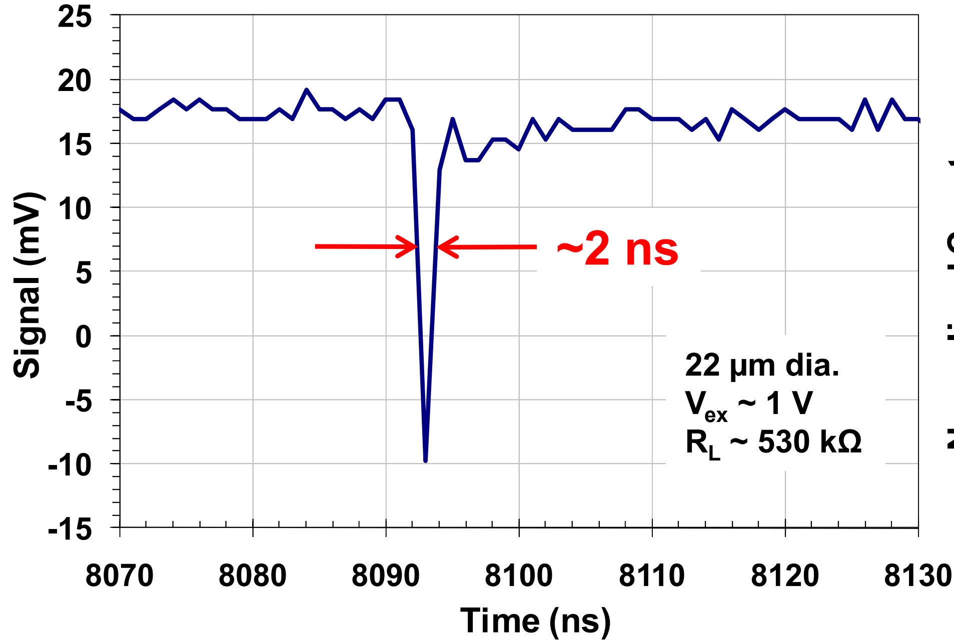

$R_L \sim 90\ k\Omega$

# Larger negative feedback provides even more effective self-quenching

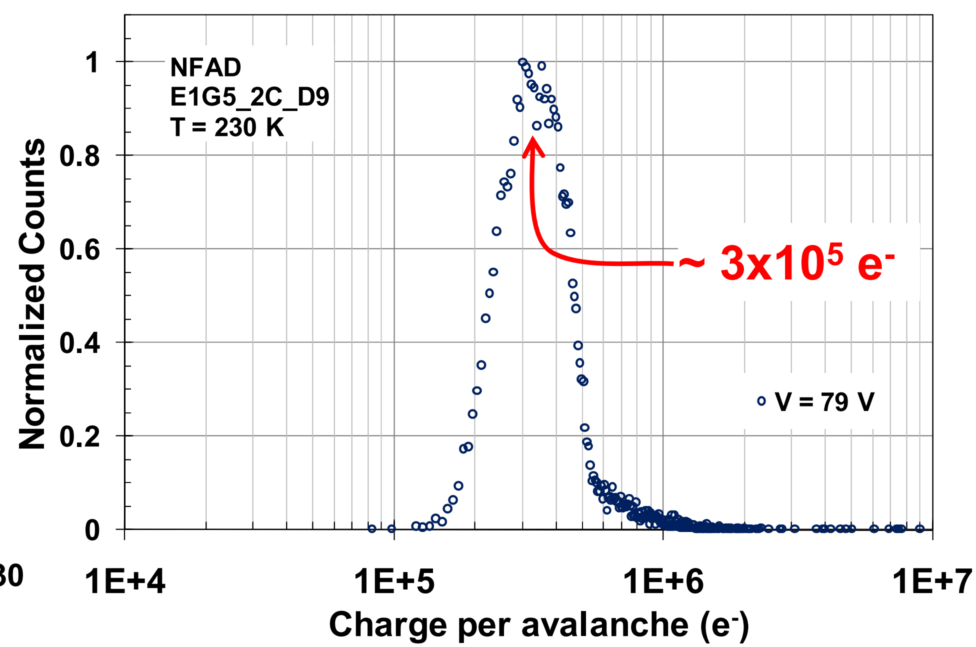
- **NFAD avalanche response: pulse width ~ 2 ns, height ~25 mV**
- **Total current flow: Q ~ 3 x $10^5$ $e^-$**
- **How reproducible are NFAD avalanche properties?**

**$R_L$ ~ 500 kΩ**

**NFAD response with larger feedback**

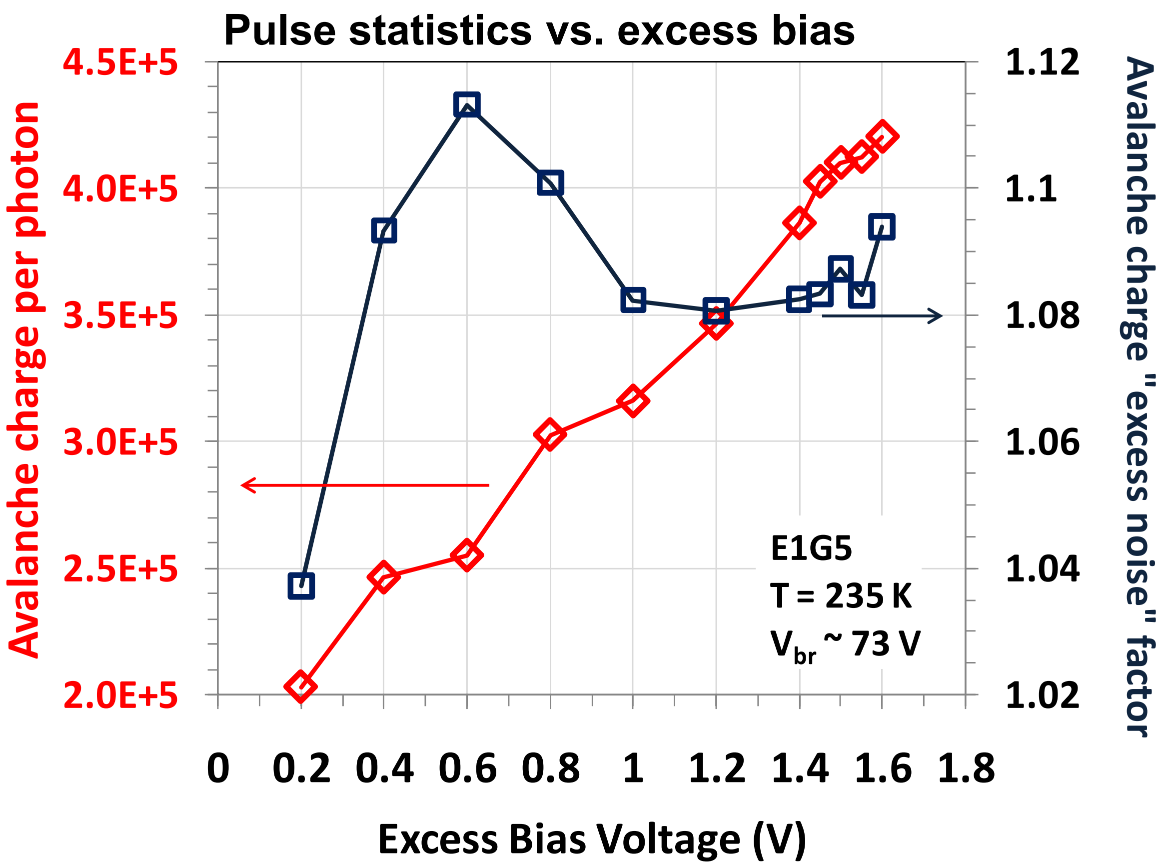



**Histogram of charge flow per avalanche**



# Statistics of avalanche charge flow

- **Analyze large number of pulses (~10,000) for pulse statistics**
- **Charge "excess noise" F(Q) is a measure of avalanche consistency**
  - Directly related to variance $\sigma^2$ of the distribution
- **Significantly more uniform avalanches than legacy Geiger-mode operation**
  - **Good prospects for resolving "summed" pulses**



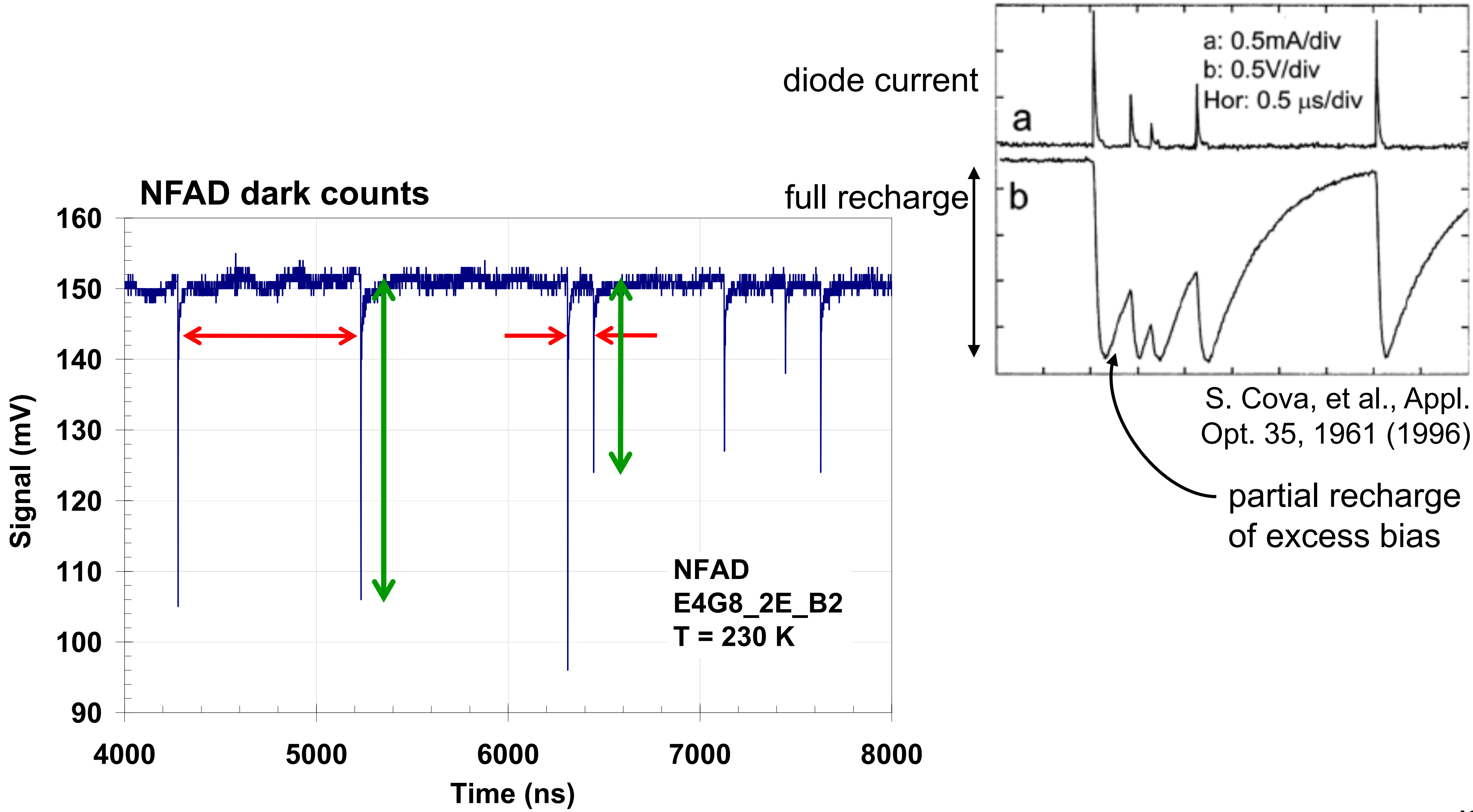

$$F(Q) = \frac{\sigma^2}{\langle Q \rangle^2} + 1$$

$\sigma / \langle Q \rangle = 0.28$

$\rightarrow F(Q) = 1.08$

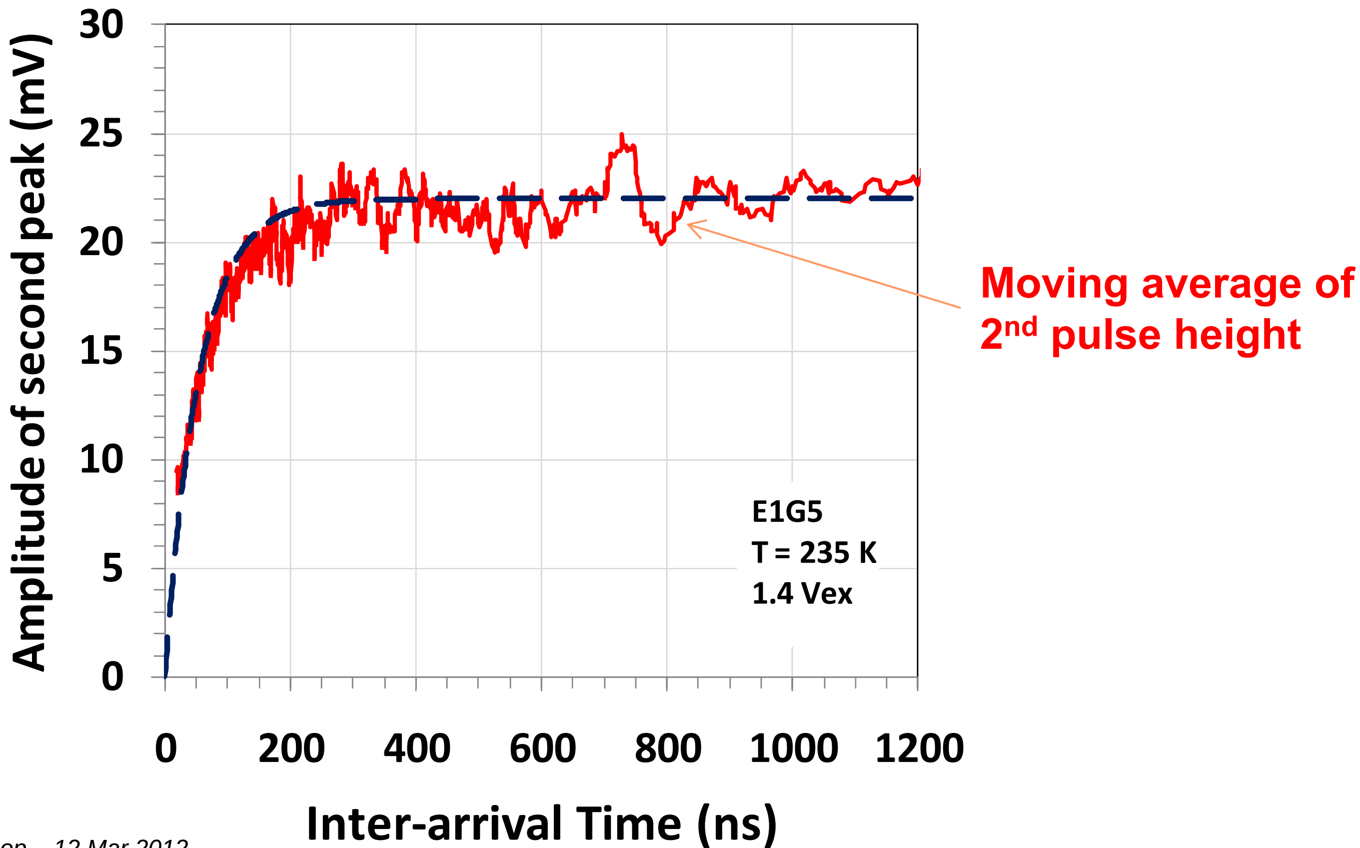
# Re-arming time from pulse height correlations

- **Look at correlation between pulse height and pulse inter-arrival time**
  - If pulse is triggered before full re-arming, pulse amplitude will tend to be lower



# Re-arming time from pulse height correlations

- Use exponential fit to 2nd pulse height (moving average) vs. interarrival time
- Find time constant $\tau$ = 55 ns; 95% recharge in $3\tau$ ~ 165 ns
- Reasonable agreement with expected $\tau = R_L C_d$ = (800 kΩ)(80 fF) ~ 64 ns



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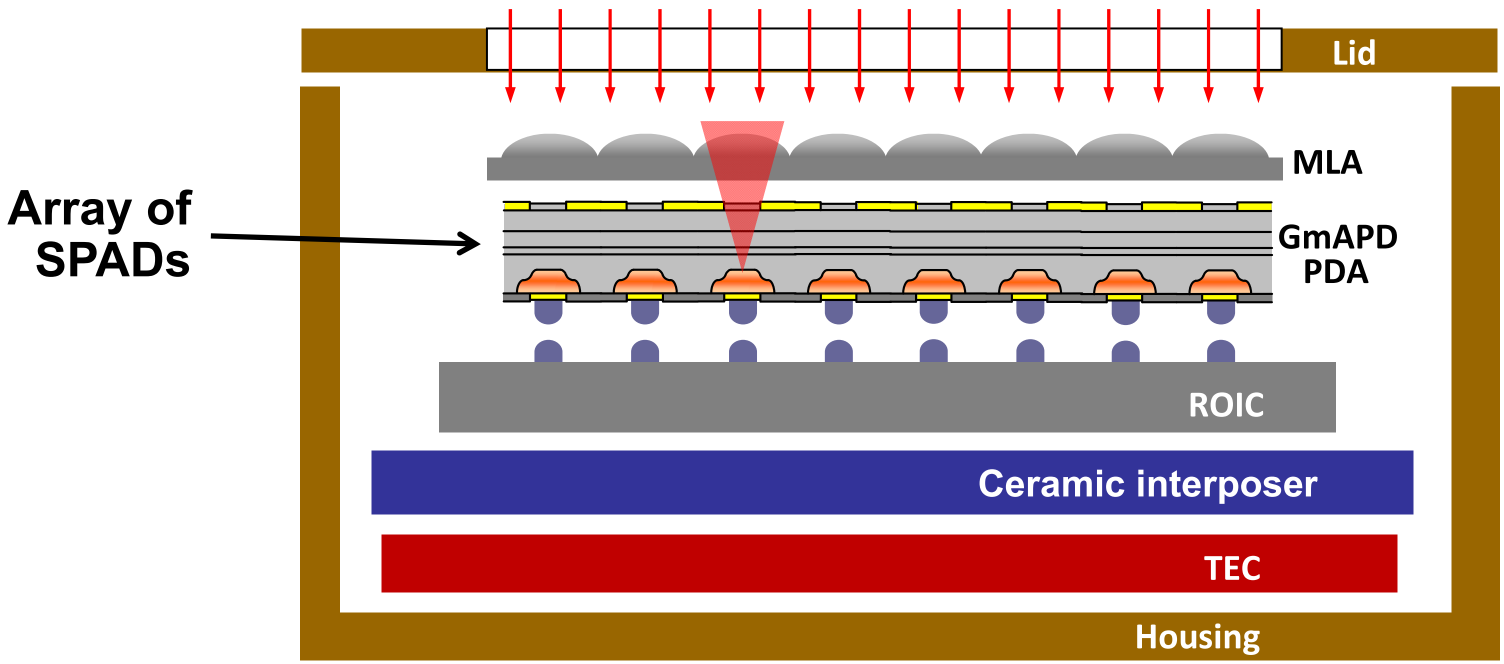
# Scaling SPADs to large-format imaging arrays

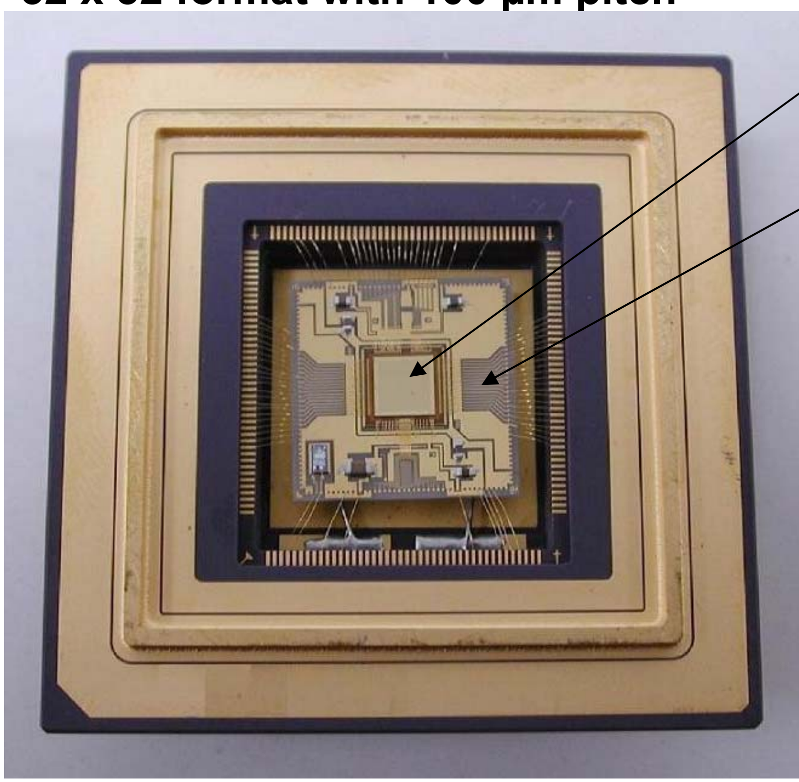
- **Focal plane array (FPA) employs three-chip stack as imaging sensor engine**
  - SPAD photodiode array (PDA)
  - CMOS readout integrated circuit (ROIC)
  - GaP microlens array (MLA)
- **Indium bump flip-chip hybridization of PDA to ROIC**
- **Passive µm-scale MLA alignment and attachment to PDA**

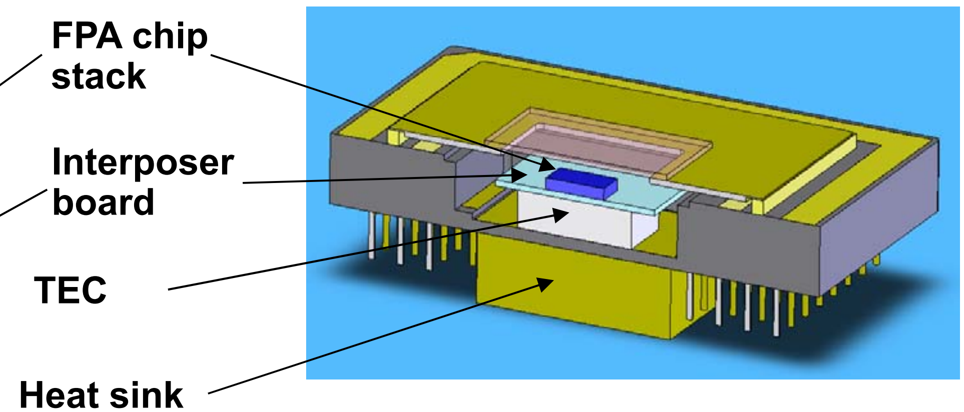


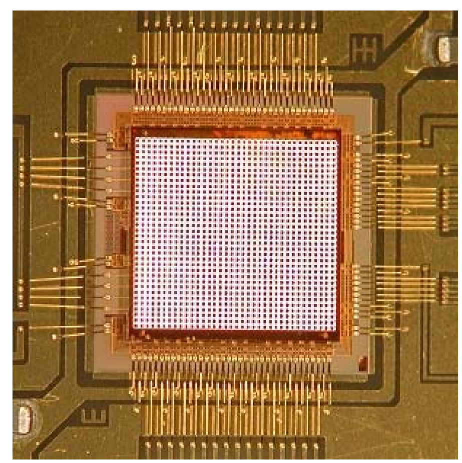
# Focal plane array module assembly

- **Manage electrical, thermal, and optical interfaces to FPA**
  - 175-connection pin grid array package
  - Thermoelectric cooler (TEC) maintains ΔT ~ 55°C with CuW heat sink
  - Microlens array on chip stack provides ~75% fill factor
  - Hermetic lid with sapphire window

**Assembled FPA module**
**32 x 32 format with 100 μm pitch**





**Chip stack on interposer**

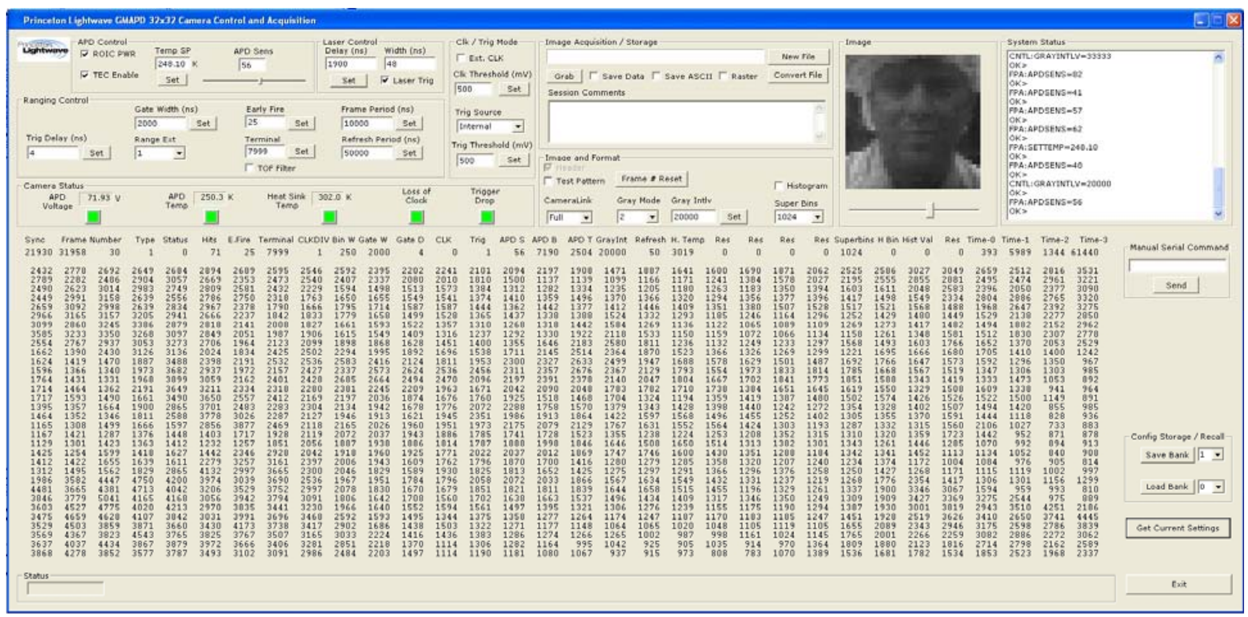
# Turn-key FPGA-driven camera system

Princeton Lightwave

- **Three-board turn-key commercial camera**
  - FPA board, FPGA board, and Interface board
- **Adjustable frame period ("range gate") between 4 ns and 10 μs**
- **32 x 32 format (100 μm pitch) with ~200,000 frames per second**


10 cm x 10 cm x 8 cm

Comprehensive GUI



# DCR & PDE performance maps

Princeton Lightwave

- **Full-camera 32 x 32 maps of DCR and PDE at 1.06 μm**
  - PDE obtained using broad illumination
- **100% pixel yield**: <u>all pixels in spec</u>

**DCR (in kHz)**

DCR Mean = 13.6 kHz
DCR σ = 2.2 kHz

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 9 | 8 | 10 | 9 | 9 | 9 | 9 | 9 | 9 | 8 | 9 | 8 | 8 | 9 | 8 | 9 | 10 | 10 | 12 | 10 | 10 | 10 | 10 | 10 | 9 | 10 | 11 | 11 | 10 | 11 | 10 | 10 |
| 2 | 8 | 10 | 10 | 10 | 9 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 11 | 11 | 10 | 12 | 10 | 12 | 12 | 11 | 12 | 11 | 11 | 13 | 11 | 12 | 12 | 10 | 11 | 9 | 9 |
| 3 | 8 | 10 | 10 | 11 | 10 | 11 | 10 | 10 | 9 | 10 | 11 | 10 | 11 | 11 | 14 | 13 | 12 | 11 | 12 | 11 | 13 | 12 | 12 | 11 | 12 | 10 | 11 | 12 | 11 | 12 | 10 | 10 |
| 4 | 9 | 9 | 9 | 10 | 10 | 11 | 11 | 10 | 11 | 10 | 11 | 10 | 13 | 12 | 14 | 11 | 13 | 11 | 13 | 12 | 13 | 12 | 13 | 12 | 14 | 12 | 11 | 13 | 11 | 11 | 11 | 11 |
| 5 | 8 | 10 | 9 | 10 | 10 | 11 | 13 | 11 | 10 | 12 | 12 | 12 | 12 | 12 | 12 | 13 | 12 | 13 | 13 | 14 | 13 | 13 | 12 | 12 | 13 | 13 | 13 | 11 | 11 | 12 | 11 | 12 |
| 6 | 8 | 10 | 10 | 10 | 8 | 10 | 12 | 11 | 12 | 12 | 11 | 13 | 12 | 12 | 14 | 13 | 14 | 14 | 13 | 13 | 12 | 13 | 13 | 14 | 13 | 14 | 12 | 13 | 11 | 13 | 11 | 11 |
| 7 | 8 | 10 | 9 | 9 | 11 | 12 | 12 | 12 | 12 | 12 | 13 | 12 | 13 | 11 | 13 | 13 | 13 | 13 | 13 | 15 | 13 | 14 | 13 | 14 | 13 | 14 | 13 | 12 | 13 | 13 | 11 | 12 |
| 8 | 9 | 9 | 10 | 12 | 11 | 11 | 11 | 11 | 13 | 12 | 12 | 13 | 13 | 13 | 12 | 13 | 13 | 13 | 13 | 14 | 13 | 13 | 13 | 14 | 14 | 13 | 13 | 15 | 13 | 12 | 11 | 11 |
| 9 | 10 | 11 | 11 | 11 | 11 | 12 | 12 | 13 | 13 | 13 | 12 | 13 | 14 | 12 | 13 | 14 | 14 | 14 | 15 | 15 | 14 | 14 | 13 | 14 | 14 | 14 | 13 | 14 | 13 | 13 | 12 | 11 |
| 10 | 9 | 10 | 11 | 11 | 12 | 11 | 12 | 13 | 12 | 12 | 13 | 13 | 13 | 14 | 12 | 13 | 15 | 14 | 13 | 14 | 13 | 15 | 14 | 14 | 14 | 13 | 15 | 13 | 14 | 14 | 13 | 11 |
| 11 | 9 | 11 | 11 | 11 | 12 | 11 | 12 | 11 | 11 | 12 | 12 | 13 | 14 | 14 | 12 | 15 | 15 | 15 | 13 | 15 | 15 | 13 | 13 | 15 | 15 | 14 | 14 | 13 | 13 | 13 | 13 | 13 |
| 12 | 11 | 11 | 12 | 12 | 12 | 11 | 12 | 13 | 13 | 13 | 11 | 13 | 14 | 14 | 13 | 15 | 14 | 15 | 15 | 15 | 15 | 15 | 14 | 13 | 14 | 14 | 14 | 13 | 14 | 13 | 12 | 12 |
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| 14 | 11 | 11 | 11 | 13 | 13 | 13 | 14 | 13 | 13 | 14 | 13 | 15 | 13 | 13 | 13 | 14 | 15 | 14 | 14 | 17 | 14 | 17 | 15 | 15 | 15 | 15 | 14 | 14 | 14 | 14 | 14 | 14 |
| 15 | 10 | 12 | 12 | 12 | 13 | 13 | 14 | 15 | 14 | 15 | 14 | 14 | 15 | 14 | 15 | 14 | 14 | 15 | 16 | 15 | 14 | 15 | 15 | 14 | 14 | 14 | 14 | 15 | 15 | 14 | 14 | 13 |
| 16 | 12 | 12 | 13 | 13 | 13 | 12 | 14 | 15 | 14 | 12 | 14 | 14 | 15 | 15 | 14 | 17 | 14 | 15 | 14 | 14 | 15 | 15 | 15 | 15 | 14 | 16 | 14 | 15 | 15 | 16 | 14 | 14 |
| 17 | 11 | 11 | 12 | 13 | 12 | 14 | 13 | 14 | 14 | 14 | 15 | 14 | 13 | 15 | 15 | 15 | 16 | 15 | 15 | 16 | 16 | 17 | 17 | 16 | 16 | 15 | 16 | 14 | 16 | 14 | 13 | 14 |
| 18 | 10 | 9 | 11 | 13 | 13 | 12 | 15 | 14 | 15 | 14 | 14 | 15 | 14 | 14 | 16 | 16 | 15 | 16 | 16 | 15 | 14 | 16 | 16 | 16 | 16 | 16 | 17 | 16 | 14 | 14 | 13 | 13 |
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| 20 | 11 | 12 | 12 | 13 | 14 | 13 | 15 | 17 | 14 | 15 | 15 | 16 | 15 | 14 | 15 | 14 | 15 | 15 | 14 | 15 | 15 | 16 | 18 | 17 | 15 | 18 | 18 | 16 | 15 | 16 | 14 | 14 |
| 21 | 9 | 12 | 13 | 14 | 14 | 13 | 14 | 15 | 15 | 15 | 14 | 16 | 16 | 17 | 15 | 16 | 15 | 14 | 15 | 17 | 15 | 15 | 16 | 18 | 17 | 18 | 16 | 17 | 16 | 15 | 15 | 14 |
| 22 | 10 | 12 | 13 | 12 | 15 | 13 | 15 | 16 | 16 | 15 | 15 | 16 | 16 | 14 | 16 | 15 | 16 | 14 | 16 | 15 | 16 | 17 | 17 | 16 | 16 | 17 | 16 | 16 | 18 | 13 | 17 | 16 |
| 23 | 11 | 11 | 12 | 12 | 14 | 16 | 15 | 14 | 15 | 15 | 15 | 16 | 14 | 13 | 17 | 15 | 16 | 15 | 16 | 16 | 17 | 16 | 15 | 17 | 16 | 18 | 16 | 16 | 16 | 16 | 15 | 14 |
| 24 | 11 | 10 | 12 | 12 | 14 | 14 | 15 | 15 | 15 | 14 | 17 | 14 | 16 | 17 | 16 | 17 | 16 | 17 | 16 | 17 | 17 | 16 | 17 | 16 | 16 | 17 | 16 | 16 | 18 | 15 | 17 | 15 |
| 25 | 11 | 11 | 12 | 13 | 14 | 16 | 14 | 15 | 15 | 16 | 15 | 16 | 17 | 16 | 15 | 16 | 16 | 16 | 16 | 17 | 15 | 16 | 16 | 17 | 17 | 19 | 17 | 17 | 16 | 16 | 15 | 15 |
| 26 | 10 | 11 | 12 | 14 | 14 | 12 | 15 | 14 | 14 | 14 | 15 | 13 | 15 | 16 | 17 | 16 | 16 | 17 | 17 | 15 | 17 | 17 | 16 | 17 | 16 | 18 | 17 | 16 | 16 | 14 | 16 | 15 |
| 27 | 11 | 11 | 12 | 13 | 15 | 14 | 15 | 16 | 14 | 17 | 15 | 16 | 15 | 17 | 15 | 17 | 18 | 17 | 17 | 15 | 17 | 17 | 16 | 18 | 16 | 17 | 16 | 16 | 16 | 15 | 15 | 14 |
| 28 | 11 | 11 | 13 | 12 | 14 | 15 | 15 | 14 | 14 | 15 | 15 | 17 | 16 | 16 | 17 | 16 | 16 | 16 | 14 | 17 | 15 | 15 | 17 | 16 | 16 | 17 | 16 | 18 | 16 | 15 | 16 | 16 |
| 29 | 11 | 12 | 12 | 13 | 13 | 15 | 13 | 14 | 14 | 14 | 15 | 15 | 15 | 16 | 16 | 16 | 16 | 16 | 16 | 15 | 16 | 17 | 17 | 16 | 17 | 16 | 16 | 15 | 16 | 16 | 15 | 14 |
| 30 | 11 | 11 | 12 | 13 | 13 | 13 | 14 | 14 | 14 | 15 | 15 | 16 | 15 | 16 | 15 | 15 | 16 | 15 | 17 | 16 | 15 | 14 | 16 | 16 | 18 | 16 | 17 | 17 | 16 | 17 | 15 | 15 |
| 31 | 10 | 12 | 11 | 11 | 12 | 12 | 13 | 13 | 13 | 13 | 13 | 14 | 15 | 14 | 16 | 14 | 15 | 14 | 14 | 14 | 14 | 14 | 15 | 14 | 16 | 15 | 13 | 15 | 15 | 15 | 14 | 16 |
| 32 | 11 | 11 | 12 | 12 | 12 | 12 | 11 | 12 | 11 | 12 | 12 | 12 | 12 | 11 | 14 | 14 | 15 | 14 | 12 | 15 | 15 | 13 | 14 | 14 | 14 | 14 | 15 | 15 | 14 | 14 | 14 | 16 |

**PDE (in %)**

PDE Mean = 39%
PDE σ = 6.3 %

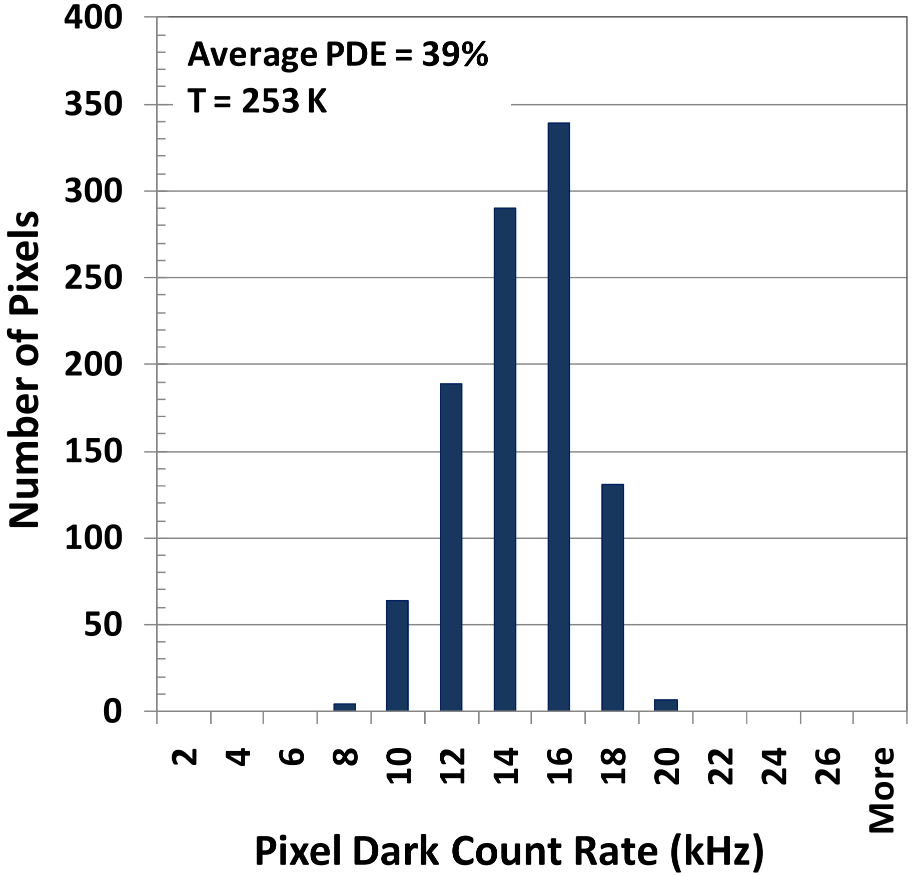
**T = 253 K**

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 28 | 23 | 27 | 27 | 30 | 26 | 29 | 30 | 26 | 27 | 27 | 29 | 23 | 25 | 25 | 27 | 25 | 28 | 26 | 28 | 24 | 27 | 27 | 25 | 28 | 29 | 27 | 24 | 27 | 25 | 25 | 27 |
| 2 | 23 | 31 | 27 | 32 | 29 | 30 | 32 | 31 | 30 | 29 | 28 | 32 | 31 | 28 | 25 | 28 | 28 | 29 | 29 | 27 | 26 | 22 | 31 | 27 | 29 | 28 | 28 | 29 | 28 | 31 | 28 | 30 |
| 3 | 30 | 28 | 29 | 32 | 31 | 29 | 34 | 32 | 31 | 32 | 32 | 34 | 33 | 33 | 31 | 28 | 31 | 30 | 30 | 29 | 27 | 27 | 27 | 31 | 29 | 32 | 31 | 27 | 30 | 29 | 26 | 29 |
| 4 | 25 | 30 | 31 | 31 | 31 | 31 | 33 | 30 | 35 | 33 | 31 | 32 | 33 | 29 | 32 | 33 | 29 | 29 | 33 | 31 | 30 | 33 | 31 | 31 | 31 | 33 | 32 | 31 | 31 | 30 | 27 | 28 |
| 5 | 28 | 27 | 27 | 30 | 31 | 33 | 32 | 31 | 37 | 33 | 35 | 33 | 31 | 31 | 32 | 27 | 30 | 34 | 32 | 31 | 33 | 30 | 32 | 31 | 31 | 31 | 29 | 32 | 31 | 29 | 29 | 31 |
| 6 | 27 | 26 | 30 | 32 | 33 | 34 | 33 | 33 | 38 | 34 | 37 | 36 | 34 | 33 | 30 | 32 | 30 | 35 | 34 | 33 | 33 | 32 | 32 | 33 | 34 | 35 | 33 | 30 | 34 | 29 | 27 | 30 |
| 7 | 29 | 30 | 29 | 30 | 29 | 33 | 36 | 38 | 33 | 35 | 34 | 38 | 34 | 33 | 33 | 35 | 35 | 31 | 33 | 28 | 36 | 31 | 36 | 32 | 34 | 35 | 35 | 34 | 34 | 28 | 29 | 30 |
| 8 | 26 | 32 | 31 | 33 | 33 | 35 | 34 | 36 | 39 | 34 | 38 | 35 | 32 | 34 | 35 | 34 | 37 | 36 | 35 | 37 | 36 | 37 | 36 | 38 | 34 | 36 | 34 | 31 | 37 | 32 | 30 | 33 |
| 9 | 31 | 34 | 34 | 33 | 31 | 33 | 34 | 36 | 37 | 33 | 37 | 34 | 35 | 37 | 34 | 36 | 36 | 40 | 37 | 39 | 39 | 36 | 36 | 34 | 37 | 35 | 37 | 34 | 34 | 30 | 30 | 32 |
| 10 | 32 | 35 | 31 | 34 | 31 | 35 | 39 | 35 | 37 | 34 | 33 | 37 | 34 | 33 | 36 | 35 | 38 | 38 | 37 | 39 | 38 | 36 | 39 | 38 | 39 | 35 | 35 | 35 | 33 | 35 | 32 | 33 |
| 11 | 29 | 32 | 32 | 31 | 35 | 35 | 37 | 38 | 39 | 35 | 40 | 37 | 39 | 40 | 38 | 35 | 39 | 41 | 40 | 37 | 37 | 37 | 39 | 38 | 39 | 39 | 34 | 37 | 35 | 33 | 32 | 32 |
| 12 | 33 | 33 | 33 | 35 | 37 | 37 | 37 | 37 | 35 | 36 | 37 | 39 | 37 | 36 | 40 | 41 | 43 | 39 | 39 | 40 | 34 | 39 | 40 | 40 | 38 | 39 | 38 | 41 | 35 | 36 | 30 | 34 |
| 13 | 31 | 34 | 34 | 34 | 38 | 38 | 40 | 39 | 36 | 37 | 40 | 35 | 39 | 37 | 40 | 39 | 40 | 38 | 42 | 37 | 43 | 41 | 38 | 42 | 41 | 39 | 36 | 39 | 38 | 36 | 32 | 34 |
| 14 | 35 | 36 | 35 | 36 | 38 | 40 | 41 | 39 | 37 | 35 | 41 | 41 | 41 | 40 | 40 | 40 | 44 | 45 | 40 | 38 | 39 | 38 | 39 | 40 | 42 | 42 | 40 | 39 | 39 | 37 | 32 | 32 |
| 15 | 34 | 31 | 38 | 37 | 39 | 41 | 45 | 37 | 38 | 34 | 41 | 38 | 40 | 40 | 42 | 38 | 44 | 39 | 40 | 42 | 40 | 39 | 39 | 42 | 42 | 43 | 45 | 41 | 40 | 37 | 34 | 33 |
| 16 | 33 | 33 | 40 | 37 | 37 | 42 | 45 | 42 | 38 | 37 | 36 | 42 | 39 | 41 | 42 | 43 | 42 | 43 | 43 | 45 | 41 | 43 | 40 | 39 | 43 | 43 | 41 | 41 | 41 | 38 | 37 | 35 |
| 17 | 38 | 36 | 38 | 37 | 39 | 44 | 42 | 40 | 40 | 38 | 36 | 43 | 44 | 43 | 41 | 43 | 42 | 41 | 44 | 44 | 42 | 37 | 39 | 43 | 39 | 42 | 44 | 44 | 39 | 39 | 36 | 35 |
| 18 | 35 | 39 | 41 | 37 | 37 | 44 | 43 | 42 | 41 | 39 | 39 | 39 | 41 | 47 | 40 | 43 | 40 | 44 | 45 | 44 | 44 | 44 | 43 | 37 | 43 | 43 | 42 | 42 | 39 | 38 | 37 | 37 |
| 19 | 35 | 39 | 37 | 38 | 41 | 40 | 43 | 40 | 43 | 41 | 38 | 41 | 43 | 42 | 46 | 43 | 45 | 43 | 44 | 44 | 48 | 45 | 42 | 44 | 41 | 42 | 41 | 42 | 43 | 42 | 34 | 37 |
| 20 | 34 | 41 | 39 | 40 | 41 | 43 | 42 | 42 | 44 | 44 | 40 | 38 | 42 | 44 | 45 | 50 | 46 | 44 | 47 | 47 | 47 | 46 | 42 | 45 | 46 | 41 | 38 | 46 | 40 | 41 | 35 | 39 |
| 21 | 37 | 40 | 38 | 41 | 40 | 45 | 41 | 43 | 40 | 44 | 41 | 45 | 43 | 42 | 43 | 44 | 42 | 47 | 45 | 44 | 47 | 47 | 48 | 45 | 45 | 43 | 41 | 43 | 38 | 43 | 35 | 37 |
| 22 | 39 | 35 | 40 | 42 | 40 | 43 | 43 | 43 | 46 | 45 | 43 | 39 | 46 | 46 | 41 | 44 | 42 | 47 | 46 | 42 | 47 | 48 | 44 | 46 | 47 | 45 | 46 | 45 | 42 | 43 | 36 | 35 |
| 23 | 34 | 37 | 41 | 42 | 41 | 40 | 44 | 43 | 47 | 44 | 49 | 44 | 47 | 45 | 42 | 47 | 45 | 45 | 46 | 45 | 42 | 44 | 48 | 48 | 45 | 48 | 44 | 44 | 43 | 46 | 39 | 41 |
| 24 | 36 | 40 | 42 | 43 | 43 | 38 | 44 | 41 | 44 | 50 | 46 | 48 | 43 | 47 | 43 | 46 | 45 | 46 | 46 | 45 | 44 | 48 | 48 | 49 | 50 | 48 | 50 | 49 | 45 | 46 | 38 | 41 |
| 25 | 34 | 38 | 42 | 43 | 40 | 43 | 45 | 45 | 44 | 44 | 47 | 43 | 46 | 47 | 50 | 50 | 44 | 46 | 51 | 44 | 49 | 52 | 51 | 48 | 52 | 47 | 48 | 46 | 45 | 45 | 38 | 45 |
| 26 | 39 | 39 | 40 | 39 | 41 | 43 | 42 | 45 | 45 | 46 | 48 | 53 | 51 | 50 | 46 | 50 | 50 | 52 | 48 | 52 | 50 | 46 | 48 | 48 | 49 | 46 | 49 | 49 | 46 | 46 | 35 | 41 |
| 27 | 37 | 38 | 45 | 39 | 46 | 43 | 42 | 42 | 46 | 43 | 47 | 45 | 50 | 46 | 48 | 46 | 47 | 50 | 46 | 50 | 52 | 52 | 49 | 48 | 51 | 44 | 48 | 46 | 46 | 44 | 38 | 42 |
| 28 | 38 | 40 | 39 | 44 | 41 | 47 | 46 | 45 | 46 | 42 | 46 | 48 | 45 | 48 | 47 | 48 | 49 | 51 | 49 | 49 | 50 | 54 | 44 | 47 | 48 | 47 | 52 | 43 | 44 | 41 | 38 | 41 |
| 29 | 37 | 38 | 40 | 40 | 40 | 43 | 44 | 42 | 47 | 42 | 42 | 39 | 43 | 49 | 46 | 46 | 50 | 48 | 47 | 46 | 50 | 48 | 45 | 49 | 46 | 49 | 38 | 47 | 43 | 44 | 42 | 43 |
| 30 | 37 | 38 | 37 | 39 | 41 | 41 | 43 | 40 | 45 | 43 | 42 | 42 | 43 | 41 | 44 | 47 | 45 | 44 | 47 | 45 | 46 | 47 | 46 | 46 | 42 | 43 | 44 | 46 | 43 | 38 | 39 | 43 |
| 31 | 38 | 35 | 37 | 40 | 39 | 43 | 40 | 40 | 42 | 45 | 42 | 43 | 42 | 43 | 43 | 48 | 44 | 45 | 43 | 46 | 44 | 44 | 43 | 47 | 41 | 41 | 47 | 42 | 42 | 43 | 39 | 41 |
| 32 | 34 | 41 | 39 | 38 | 42 | 38 | 43 | 40 | 46 | 44 | 44 | 43 | 42 | 45 | 41 | 43 | 41 | 43 | 44 | 43 | 43 | 43 | 43 | 44 | 38 | 43 | 43 | 41 | 44 | 42 | 40 | 40 |

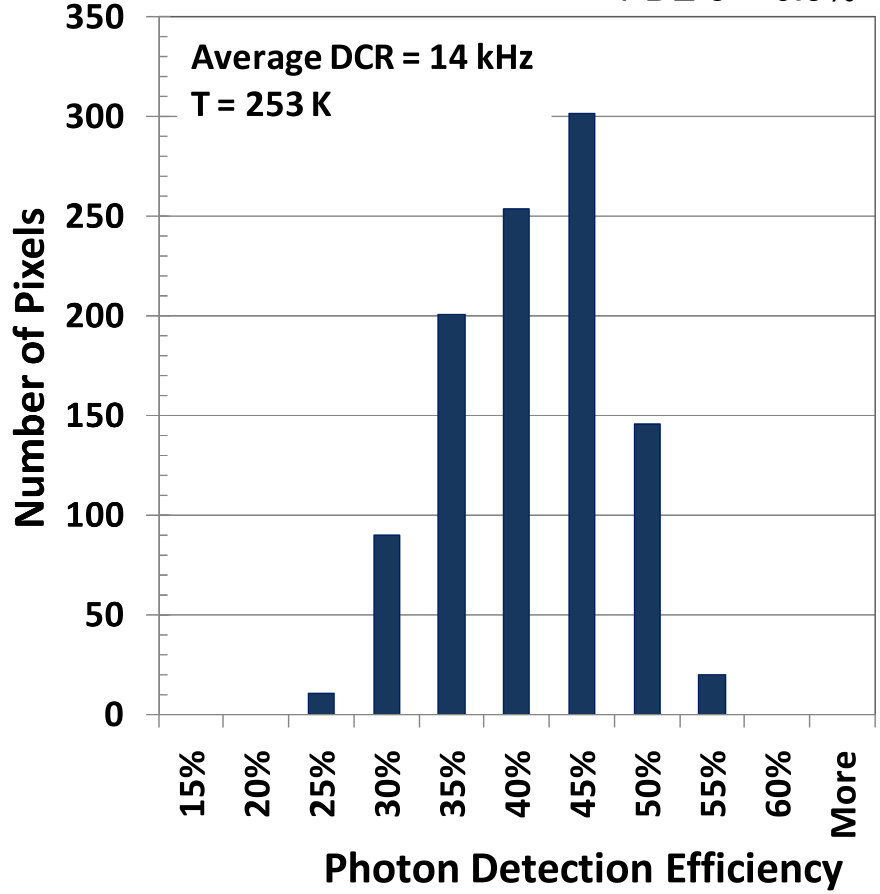
# DCR & PDE distributions

- **All 1024 pixels have DCR < 20 kHz for mean PDE = 39%**

DCR Mean = 13.6 kHz
DCR σ = 2.2 kHz




PDE Mean = 39%
PDE σ = 6.3%



# Excellent low DCR demonstrated

- Average DCR as low as 2.0 kHz at 37% PDE

## DCR (in kHz)

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.0 | 1.4 | 0.0 | 0.0 | 0.0 | 0.0 | 1.5 | 1.0 | 1.5 | 1.4 | 1.6 | 1.1 | 1.0 | 1.3 | 1.2 | 1.5 | 0.8 | 1.4 | 1.5 | 1.0 | 1.4 | 1.4 | 1.4 | 1.4 | 1.6 | 1.3 | 1.4 | 1.8 | 1.6 | 1.4 | 1.8 | 1.5 |
| 2 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 1.8 | 1.7 | 1.6 | 1.2 | 1.3 | 1.5 | 1.5 | 2.1 | 1.6 | 1.6 | 1.4 | 1.3 | 1.6 | 1.4 | 1.5 | 1.0 | 1.4 | 2.3 | 1.8 | 1.9 | 1.5 | 1.6 | 2.0 | 1.1 | 1.4 | 1.6 | 1.7 |
| 3 | 0.0 | 0.0 | 0.0 | 1.9 | 0.8 | 2.0 | 1.7 | 1.8 | 1.6 | 1.5 | 1.8 | 1.8 | 1.7 | 1.4 | 1.5 | 1.5 | 1.5 | 1.7 | 1.8 | 1.3 | 1.3 | 2.1 | 1.8 | 1.6 | 2.0 | 2.0 | 1.7 | 1.8 | 1.5 | 1.8 | 1.9 | 1.6 |
| 4 | 0.1 | 1.8 | 0.9 | 1.7 | 2.0 | 1.5 | 1.8 | 1.9 | 1.8 | 1.8 | 1.6 | 2.1 | 2.2 | 1.7 | 1.6 | 1.7 | 1.3 | 1.8 | 1.8 | 2.0 | 1.7 | 2.1 | 2.2 | 1.8 | 1.7 | 2.6 | 2.1 | 1.8 | 1.6 | 1.8 | 1.5 | 1.6 |
| 5 | 0.0 | 2.2 | 1.7 | 1.7 | 1.6 | 1.9 | 1.7 | 2.2 | 1.8 | 2.0 | 1.6 | 1.6 | 1.8 | 1.8 | 2.1 | 1.8 | 1.6 | 2.0 | 2.0 | 1.7 | 1.8 | 1.7 | 1.8 | 1.5 | 2.1 | 1.7 | 1.8 | 1.8 | 1.2 | 1.8 | 2.2 | 1.6 |
| 6 | 1.2 | 0.1 | 1.7 | 2.1 | 2.4 | 1.7 | 1.6 | 1.7 | 1.5 | 1.9 | 1.9 | 1.3 | 1.8 | 1.5 | 1.8 | 1.6 | 1.2 | 1.5 | 1.6 | 1.8 | 1.4 | 1.9 | 2.0 | 1.7 | 1.8 | 2.3 | 2.2 | 1.7 | 2.0 | 2.4 | 1.9 | 1.5 |
| 7 | 1.0 | 1.7 | 1.4 | 2.3 | 1.7 | 2.1 | 1.8 | 2.3 | 2.0 | 1.8 | 1.9 | 1.8 | 1.7 | 1.6 | 1.9 | 1.6 | 1.7 | 2.2 | 1.7 | 1.3 | 1.6 | 1.8 | 2.2 | 2.2 | 1.3 | 2.0 | 1.8 | 2.2 | 1.8 | 2.1 | 2.2 | 2.2 |
| 8 | 1.2 | 1.2 | 1.6 | 2.4 | 1.8 | 1.7 | 2.2 | 2.4 | 1.5 | 2.2 | 1.8 | 1.9 | 1.8 | 1.7 | 1.5 | 2.2 | 1.3 | 2.0 | 1.5 | 1.9 | 1.8 | 1.7 | 2.0 | 1.6 | 1.9 | 2.1 | 1.9 | 1.9 | 2.4 | 1.8 | 1.9 | 1.4 |
| 9 | 1.8 | 1.8 | 1.4 | 2.0 | 1.6 | 2.1 | 1.6 | 1.8 | 1.9 | 1.4 | 1.6 | 1.7 | 2.0 | 2.3 | 1.7 | 1.8 | 1.6 | 1.7 | 1.7 | 1.5 | 1.6 | 1.9 | 2.1 | 2.6 | 1.8 | 1.9 | 2.0 | 1.3 | 2.0 | 1.8 | 1.3 | 1.8 |
| 10 | 1.8 | 2.1 | 1.6 | 1.6 | 1.6 | 1.8 | 1.5 | 1.6 | 1.8 | 1.6 | 1.7 | 1.4 | 2.0 | 1.7 | 2.2 | 1.9 | 1.7 | 1.9 | 1.8 | 2.0 | 2.1 | 1.7 | 1.9 | 2.1 | 1.4 | 2.0 | 2.0 | 2.0 | 1.4 | 2.0 | 2.0 | 1.8 |
| 11 | 1.6 | 2.0 | 1.5 | 1.3 | 1.4 | 2.0 | 1.4 | 1.7 | 1.7 | 1.6 | 2.0 | 2.0 | 2.1 | 1.7 | 2.0 | 1.8 | 2.1 | 1.7 | 1.7 | 1.6 | 2.2 | 2.2 | 2.0 | 1.6 | 2.4 | 1.7 | 1.7 | 1.6 | 1.9 | 1.5 | 1.9 | 1.1 |
| 12 | 1.6 | 1.6 | 1.9 | 1.7 | 1.9 | 1.9 | 2.2 | 1.6 | 1.7 | 2.5 | 1.9 | 1.8 | 1.7 | 1.8 | 2.4 | 1.8 | 1.6 | 1.5 | 1.7 | 2.0 | 2.3 | 2.0 | 1.9 | 2.0 | 2.3 | 2.0 | 1.7 | 1.7 | 1.6 | 1.3 | 1.7 | 1.2 |
| 13 | 1.2 | 1.1 | 2.1 | 2.0 | 2.0 | 1.7 | 1.7 | 2.0 | 2.1 | 1.7 | 1.7 | 1.8 | 1.9 | 1.7 | 1.3 | 1.5 | 1.8 | 1.4 | 2.2 | 1.6 | 2.4 | 1.8 | 2.0 | 2.6 | 1.9 | 1.7 | 1.9 | 1.7 | 1.8 | 1.9 | 1.6 | 1.3 |
| 14 | 1.3 | 2.0 | 2.0 | 2.3 | 2.2 | 2.9 | 2.0 | 1.9 | 1.6 | 2.0 | 1.9 | 1.8 | 1.8 | 1.7 | 2.0 | 1.8 | 1.8 | 2.6 | 2.2 | 1.9 | 1.4 | 2.2 | 1.9 | 2.2 | 2.1 | 2.1 | 1.6 | 1.7 | 1.4 | 2.2 | 1.7 | 1.8 |
| 15 | 1.2 | 1.8 | 1.5 | 1.8 | 2.6 | 2.1 | 2.2 | 2.1 | 2.3 | 1.7 | 2.0 | 1.9 | 2.0 | 2.3 | 1.8 | 1.9 | 1.8 | 1.8 | 1.5 | 2.1 | 1.8 | 2.1 | 2.2 | 2.4 | 2.2 | 2.4 | 2.3 | 2.1 | 2.1 | 1.9 | 2.2 | 1.9 |
| 16 | 2.0 | 2.0 | 2.1 | 1.7 | 2.6 | 2.4 | 2.2 | 1.8 | 2.1 | 2.4 | 1.7 | 2.0 | 2.3 | 1.9 | 2.0 | 2.1 | 2.7 | 2.6 | 2.1 | 2.3 | 2.2 | 2.1 | 1.7 | 2.0 | 1.7 | 1.5 | 2.2 | 2.0 | 1.4 | 2.3 | 1.6 | 1.7 |
| 17 | 1.8 | 2.5 | 2.2 | 2.2 | 1.8 | 2.0 | 2.3 | 2.3 | 2.7 | 2.7 | 2.0 | 2.3 | 2.0 | 2.2 | 1.9 | 2.3 | 2.4 | 1.9 | 2.2 | 2.2 | 2.3 | 2.0 | 2.3 | 2.2 | 2.0 | 2.2 | 2.0 | 1.8 | 2.0 | 2.0 | 2.1 | 2.0 |
| 18 | 1.6 | 2.4 | 2.4 | 2.2 | 2.6 | 2.1 | 2.5 | 2.2 | 2.1 | 2.0 | 1.9 | 2.3 | 2.3 | 2.6 | 2.4 | 2.0 | 1.8 | 2.2 | 1.7 | 2.4 | 2.6 | 2.3 | 2.4 | 2.7 | 2.6 | 1.8 | 2.4 | 2.0 | 2.1 | 2.6 | 1.6 | 1.8 |
| 19 | 1.7 | 2.2 | 2.4 | 2.5 | 2.2 | 1.9 | 2.6 | 2.1 | 2.1 | 2.1 | 2.6 | 2.2 | 2.4 | 2.1 | 2.7 | 2.5 | 1.2 | 2.8 | 2.2 | 2.0 | 1.8 | 2.6 | 1.9 | 2.4 | 2.2 | 1.9 | 2.0 | 2.1 | 2.7 | 2.2 | 1.8 | 1.5 |
| 20 | 2.3 | 2.1 | 2.0 | 2.2 | 2.4 | 2.2 | 2.3 | 2.1 | 1.8 | 2.2 | 2.5 | 2.3 | 3.0 | 2.6 | 2.4 | 2.4 | 2.5 | 2.4 | 3.0 | 2.3 | 2.8 | 2.8 | 2.3 | 2.7 | 2.0 | 1.9 | 1.5 | 1.5 | 2.2 | 1.7 | 2.2 | 1.8 |
| 21 | 1.6 | 1.7 | 1.8 | 2.9 | 2.0 | 2.3 | 2.0 | 2.5 | 2.4 | 2.4 | 2.2 | 2.0 | 2.0 | 2.5 | 2.7 | 1.8 | 2.6 | 2.1 | 1.9 | 2.0 | 2.3 | 2.2 | 2.6 | 2.5 | 2.6 | 1.8 | 2.0 | 1.7 | 1.4 | 1.8 | 1.8 | 2.1 |
| 22 | 2.4 | 2.3 | 2.4 | 2.6 | 2.2 | 2.1 | 2.2 | 2.6 | 2.1 | 2.5 | 2.3 | 2.6 | 2.5 | 2.4 | 2.4 | 2.6 | 2.0 | 2.1 | 2.0 | 2.4 | 2.2 | 2.2 | 1.8 | 2.9 | 1.9 | 2.6 | 1.9 | 2.0 | 2.1 | 2.4 | 1.6 | 1.7 |
| 23 | 1.6 | 1.9 | 2.4 | 2.2 | 2.4 | 2.2 | 2.1 | 2.3 | 2.4 | 1.9 | 2.6 | 2.2 | 2.1 | 2.6 | 3.1 | 1.9 | 2.8 | 2.3 | 2.0 | 1.8 | 2.5 | 2.2 | 2.0 | 2.0 | 2.3 | 2.1 | 2.2 | 2.1 | 1.5 | 2.4 | 1.7 | 1.6 |
| 24 | 1.9 | 2.4 | 2.1 | 2.4 | 2.2 | 2.2 | 2.0 | 2.3 | 2.0 | 2.4 | 2.4 | 2.2 | 1.9 | 2.3 | 2.7 | 1.6 | 2.0 | 2.1 | 2.2 | 2.0 | 1.7 | 1.8 | 1.6 | 2.1 | 2.0 | 2.4 | 2.3 | 2.6 | 2.3 | 2.2 | 2.2 | 1.8 |
| 25 | 2.1 | 1.9 | 2.1 | 1.9 | 2.1 | 2.1 | 2.4 | 2.3 | 2.2 | 2.6 | 2.4 | 2.2 | 3.0 | 2.4 | 1.9 | 1.7 | 2.6 | 2.3 | 1.8 | 2.0 | 2.0 | 2.4 | 2.8 | 2.2 | 2.2 | 2.0 | 2.2 | 2.1 | 2.3 | 2.4 | 2.4 | 1.9 |
| 26 | 2.0 | 2.0 | 2.5 | 2.8 | 2.4 | 3.0 | 1.8 | 2.4 | 2.4 | 2.4 | 1.8 | 2.1 | 3.2 | 2.4 | 2.2 | 1.8 | 2.3 | 2.2 | 2.6 | 2.7 | 2.3 | 2.2 | 2.0 | 2.1 | 2.1 | 2.8 | 2.5 | 1.8 | 2.5 | 2.1 | 1.9 | 2.3 |
| 27 | 1.9 | 1.7 | 1.9 | 2.4 | 2.2 | 2.7 | 1.9 | 2.8 | 1.8 | 2.3 | 2.4 | 2.4 | 2.4 | 2.4 | 1.8 | 1.7 | 2.4 | 1.8 | 1.9 | 2.2 | 2.1 | 2.2 | 2.6 | 2.2 | 1.9 | 2.2 | 2.5 | 2.4 | 2.4 | 1.8 | 2.0 | 1.6 |
| 28 | 1.2 | 2.0 | 2.6 | 2.5 | 1.8 | 2.9 | 1.7 | 2.4 | 2.3 | 2.2 | 2.5 | 2.6 | 1.9 | 3.4 | 2.2 | 2.5 | 2.5 | 2.0 | 2.5 | 1.8 | 2.0 | 2.4 | 2.0 | 2.7 | 2.4 | 2.0 | 2.6 | 2.4 | 1.9 | 2.1 | 1.8 | 2.0 |
| 29 | 1.2 | 2.0 | 2.4 | 2.7 | 2.4 | 2.0 | 2.0 | 2.4 | 2.0 | 2.4 | 2.1 | 1.6 | 2.3 | 2.1 | 2.3 | 2.3 | 2.0 | 2.2 | 2.0 | 2.1 | 2.2 | 1.6 | 1.9 | 2.3 | 1.8 | 2.0 | 1.9 | 2.2 | 1.8 | 2.2 | 2.0 | 1.8 |
| 30 | 1.5 | 2.0 | 1.8 | 2.2 | 2.0 | 2.0 | 2.0 | 2.2 | 2.1 | 1.8 | 1.5 | 2.9 | 2.5 | 2.0 | 1.9 | 2.3 | 2.0 | 2.1 | 2.5 | 2.1 | 2.4 | 1.8 | 1.9 | 1.9 | 1.8 | 1.6 | 2.4 | 1.8 | 1.8 | 2.3 | 1.9 | 1.3 |
| 31 | 1.7 | 2.0 | 1.4 | 2.1 | 2.0 | 1.6 | 2.2 | 2.0 | 1.7 | 1.8 | 1.7 | 1.9 | 2.1 | 1.9 | 2.2 | 2.0 | 1.9 | 1.1 | 2.3 | 1.6 | 1.8 | 1.9 | 1.8 | 1.4 | 2.2 | 1.6 | 1.5 | 1.8 | 1.7 | 2.0 | 1.9 | 1.3 |
| 32 | 1.6 | 1.2 | 1.4 | 1.2 | 1.9 | 1.6 | 1.8 | 1.8 | 2.2 | 1.6 | 1.8 | 1.7 | 1.7 | 1.7 | 1.7 | 1.6 | 1.8 | 1.5 | 1.2 | 1.2 | 1.9 | 1.4 | 1.8 | 1.3 | 1.7 | 1.4 | 1.4 | 1.3 | 1.2 | 1.4 | 1.0 | 1.6 |

## PDE (in %)

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 28 | 0 | 0 | 0 | 0 | 32 | 35 | 37 | 36 | 35 | 33 | 35 | 35 | 35 | 35 | 35 | 33 | 32 | 34 | 35 | 34 | 34 | 34 | 35 | 35 | 34 | 36 | 33 | 34 | 34 | 34 |
| 2 | 0 | 0 | 0 | 0 | 0 | 34 | 37 | 37 | 39 | 39 | 38 | 37 | 38 | 37 | 37 | 37 | 35 | 35 | 34 | 36 | 37 | 38 | 37 | 38 | 36 | 41 | 38 | 37 | 39 | 40 | 35 | 34 |
| 3 | 0 | 0 | 0 | 33 | 11 | 35 | 36 | 38 | 37 | 41 | 36 | 38 | 37 | 38 | 36 | 38 | 38 | 35 | 36 | 35 | 38 | 38 | 40 | 38 | 37 | 38 | 41 | 39 | 39 | 39 | 39 | 36 |
| 4 | 0 | 30 | 13 | 34 | 35 | 35 | 39 | 40 | 39 | 42 | 38 | 37 | 38 | 39 | 39 | 37 | 35 | 35 | 37 | 37 | 37 | 36 | 40 | 39 | 42 | 39 | 39 | 37 | 40 | 40 | 37 | 31 |
| 5 | 0 | 34 | 33 | 36 | 34 | 37 | 38 | 39 | 40 | 39 | 38 | 38 | 39 | 38 | 37 | 40 | 37 | 39 | 37 | 36 | 38 | 38 | 40 | 41 | 39 | 38 | 38 | 41 | 40 | 42 | 36 | 35 |
| 6 | 28 | 0 | 31 | 36 | 37 | 35 | 37 | 43 | 35 | 38 | 35 | 39 | 36 | 38 | 39 | 36 | 39 | 34 | 38 | 35 | 36 | 38 | 41 | 39 | 43 | 39 | 40 | 40 | 40 | 38 | 37 | 36 |
| 7 | 28 | 30 | 32 | 36 | 34 | 34 | 36 | 37 | 36 | 35 | 34 | 36 | 34 | 41 | 35 | 34 | 35 | 34 | 34 | 34 | 35 | 37 | 40 | 43 | 39 | 40 | 41 | 38 | 37 | 37 | 36 | 33 |
| 8 | 30 | 32 | 31 | 34 | 33 | 32 | 34 | 36 | 34 | 34 | 35 | 36 | 39 | 38 | 41 | 40 | 40 | 37 | 36 | 34 | 37 | 39 | 40 | 39 | 42 | 39 | 38 | 41 | 37 | 36 | 41 | 35 |
| 9 | 32 | 31 | 33 | 32 | 33 | 32 | 36 | 35 | 34 | 35 | 34 | 35 | 35 | 36 | 39 | 39 | 39 | 36 | 37 | 35 | 36 | 38 | 40 | 37 | 41 | 39 | 38 | 38 | 38 | 39 | 36 | 34 |
| 10 | 30 | 34 | 32 | 30 | 33 | 34 | 34 | 35 | 33 | 36 | 39 | 35 | 35 | 37 | 35 | 40 | 37 | 41 | 39 | 37 | 36 | 36 | 39 | 34 | 38 | 35 | 41 | 38 | 40 | 36 | 38 | 32 |
| 11 | 31 | 35 | 33 | 35 | 34 | 32 | 32 | 36 | 34 | 37 | 36 | 35 | 35 | 33 | 36 | 37 | 37 | 36 | 38 | 36 | 34 | 37 | 39 | 40 | 36 | 34 | 33 | 41 | 39 | 39 | 37 | 33 |
| 12 | 30 | 30 | 35 | 37 | 36 | 35 | 32 | 34 | 34 | 38 | 33 | 36 | 41 | 35 | 37 | 35 | 35 | 38 | 37 | 37 | 34 | 37 | 35 | 42 | 40 | 33 | 36 | 40 | 39 | 39 | 36 | 34 |
| 13 | 32 | 35 | 36 | 35 | 33 | 35 | 35 | 36 | 38 | 36 | 36 | 33 | 40 | 38 | 34 | 38 | 37 | 37 | 39 | 39 | 35 | 36 | 33 | 39 | 37 | 34 | 35 | 37 | 37 | 37 | 38 | 36 |
| 14 | 31 | 34 | 38 | 38 | 37 | 38 | 38 | 36 | 36 | 36 | 36 | 37 | 36 | 36 | 37 | 36 | 34 | 35 | 42 | 37 | 36 | 32 | 36 | 36 | 35 | 37 | 40 | 37 | 39 | 37 | 39 | 37 |
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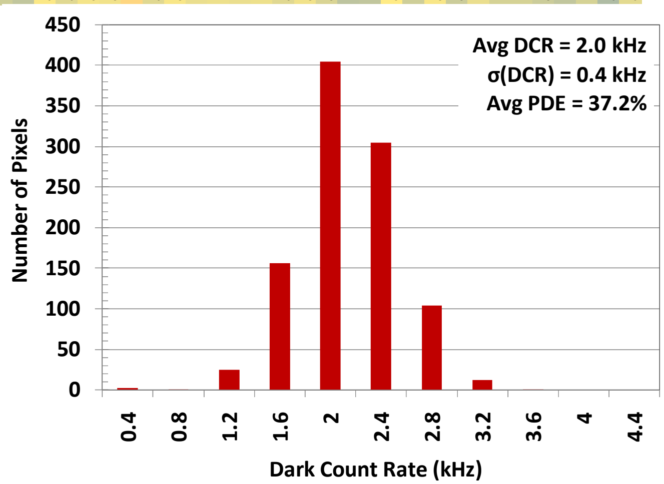
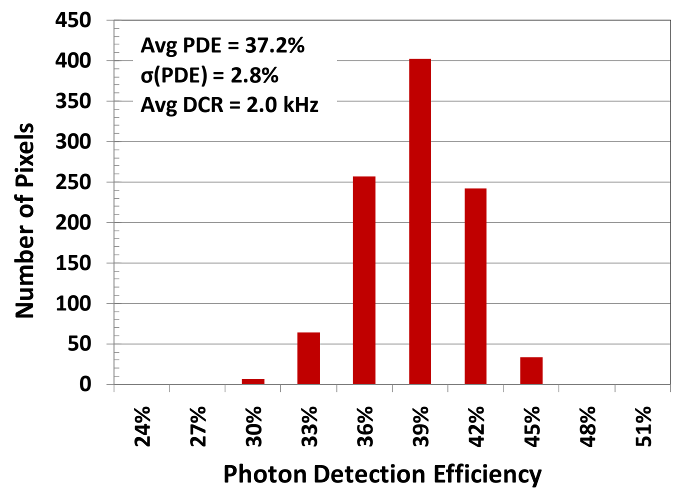
| 16 | 33 | 36 | 35 | 36 | 40 | 43 | 39 | 42 | 39 | 38 | 38 | 36 | 38 | 36 | 36 | 36 | 38 | 39 | 40 | 41 | 42 | 39 | 42 | 40 | 38 | 40 | 42 | 38 | 40 | 37 | 38 | 37 |
| 17 | 33 | 33 | 36 | 35 | 39 | 42 | 42 | 42 | 40 | 41 | 36 | 39 | 39 | 41 | 40 | 42 | 43 | 41 | 40 | 43 | 42 | 37 | 36 | 41 | 37 | 37 | 38 | 37 | 40 | 42 | 39 | 38 |
| 18 | 34 | 35 | 34 | 37 | 37 | 39 | 40 | 42 | 40 | 40 | 38 | 38 | 43 | 42 | 39 | 40 | 41 | 41 | 43 | 40 | 37 | 40 | 34 | 41 | 39 | 39 | 35 | 39 | 38 | 39 | 40 | 37 |
| 19 | 35 | 32 | 36 | 35 | 39 | 40 | 37 | 42 | 41 | 42 | 39 | 37 | 41 | 41 | 38 | 39 | 38 | 41 | 38 | 38 | 38 | 37 | 38 | 42 | 41 | 40 | 39 | 39 | 37 | 39 | 39 | 37 |
| 20 | 36 | 37 | 39 | 35 | 37 | 40 | 39 | 39 | 38 | 39 | 37 | 38 | 42 | 41 | 39 | 39 | 34 | 37 | 37 | 39 | 35 | 38 | 40 | 40 | 42 | 40 | 35 | 39 | 41 | 39 | 39 | 36 |
| 21 | 37 | 38 | 36 | 37 | 37 | 36 | 38 | 37 | 41 | 39 | 37 | 40 | 42 | 39 | 40 | 40 | 43 | 42 | 42 | 37 | 43 | 39 | 40 | 38 | 39 | 33 | 32 | 43 | 41 | 38 | 42 | 37 |
| 22 | 32 | 36 | 33 | 37 | 39 | 39 | 36 | 38 | 40 | 41 | 41 | 38 | 41 | 36 | 41 | 40 | 38 | 40 | 44 | 40 | 38 | 42 | 40 | 43 | 39 | 33 | 38 | 40 | 38 | 40 | 40 | 38 |
| 23 | 35 | 39 | 36 | 38 | 38 | 37 | 40 | 39 | 40 | 38 | 40 | 42 | 43 | 39 | 39 | 42 | 39 | 39 | 38 | 40 | 39 | 40 | 40 | 39 | 31 | 33 | 41 | 38 | 38 | 39 | 41 | 39 |
| 24 | 35 | 35 | 37 | 41 | 42 | 35 | 40 | 37 | 44 | 38 | 39 | 38 | 40 | 38 | 39 | 39 | 36 | 37 | 41 | 41 | 41 | 35 | 40 | 34 | 40 | 40 | 41 | 41 | 37 | 39 | 38 | 35 |
| 25 | 35 | 36 | 39 | 39 | 38 | 37 | 37 | 38 | 39 | 38 | 38 | 37 | 37 | 44 | 39 | 39 | 37 | 39 | 37 | 40 | 41 | 38 | 34 | 38 | 39 | 40 | 38 | 39 | 38 | 40 | 39 | 37 |
| 26 | 33 | 36 | 37 | 34 | 37 | 36 | 34 | 36 | 38 | 36 | 35 | 36 | 41 | 40 | 42 | 38 | 39 | 40 | 37 | 39 | 34 | 36 | 41 | 42 | 43 | 39 | 39 | 42 | 39 | 41 | 40 | 34 |
| 27 | 34 | 37 | 37 | 35 | 37 | 34 | 36 | 34 | 39 | 37 | 34 | 36 | 36 | 38 | 38 | 39 | 41 | 34 | 39 | 37 | 39 | 37 | 42 | 39 | 40 | 39 | 42 | 39 | 39 | 39 | 40 | 38 |
| 28 | 32 | 33 | 34 | 33 | 37 | 36 | 38 | 38 | 39 | 36 | 35 | 34 | 38 | 33 | 33 | 35 | 33 | 38 | 40 | 39 | 42 | 38 | 40 | 41 | 43 | 39 | 37 | 36 | 37 | 40 | 37 | 36 |
| 29 | 32 | 33 | 36 | 35 | 34 | 38 | 38 | 40 | 34 | 36 | 38 | 39 | 38 | 40 | 38 | 39 | 35 | 38 | 39 | 37 | 39 | 42 | 39 | 39 | 42 | 37 | 38 | 41 | 38 | 38 | 37 | 36 |
| 30 | 31 | 34 | 34 | 34 | 37 | 36 | 37 | 36 | 37 | 40 | 37 | 39 | 39 | 38 | 36 | 38 | 40 | 39 | 39 | 37 | 41 | 38 | 40 | 39 | 40 | 40 | 37 | 37 | 37 | 36 | 35 | 33 |
| 31 | 31 | 31 | 31 | 35 | 34 | 36 | 34 | 36 | 40 | 38 | 38 | 39 | 36 | 36 | 36 | 37 | 37 | 38 | 37 | 36 | 37 | 39 | 33 | 35 | 38 | 35 | 38 | 37 | 36 | 36 | 34 | 34 |
| 32 | 31 | 30 | 31 | 31 | 32 | 33 | 30 | 35 | 35 | 34 | 35 | 35 | 34 | 33 | 34 | 35 | 34 | 33 | 36 | 32 | 34 | 33 | 33 | 34 | 33 | 36 | 35 | 34 | 35 | 34 | 34 | 33 |

T = 253 K

DCR Mean = 2.0 kHz
DCR σ = 0.4 kHz

PDE Mean = 37.2%
PDE σ = 2.8%

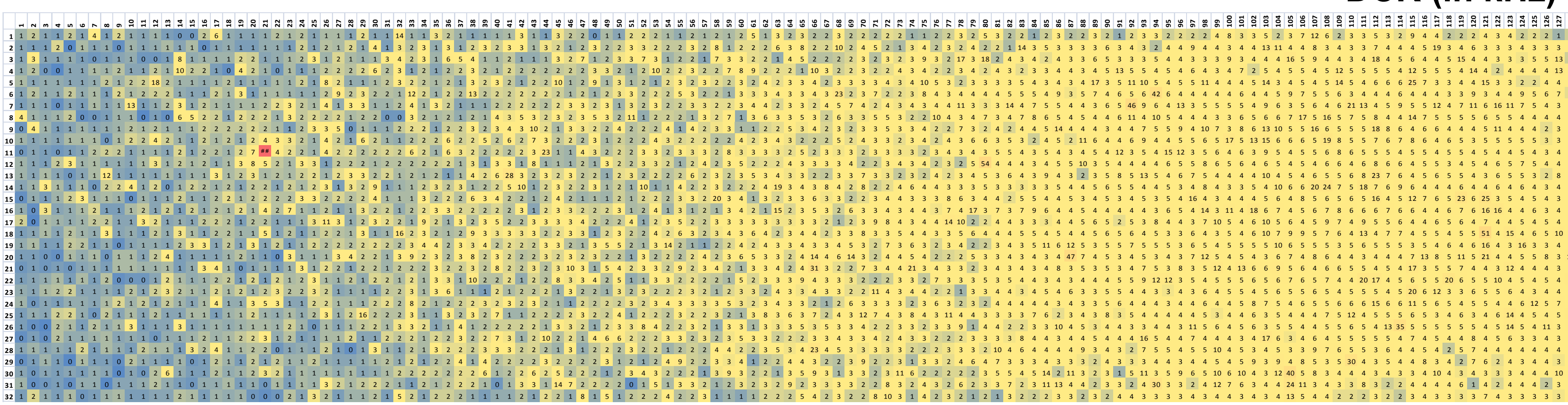


# 1.5 µm FPAs and larger format FPAs

- **32 x 32 format camera for 1.5 µm at same quality as 1.06 µm**
  - 100% pixel yield
  - Higher DCR due to narrower bandgap (InGaAs) absorber
- **32 x 128 format (50 µm pitch) at 1.06 µm with >99.9% yield**
  - Extent of performance gradient depends on location on wafer

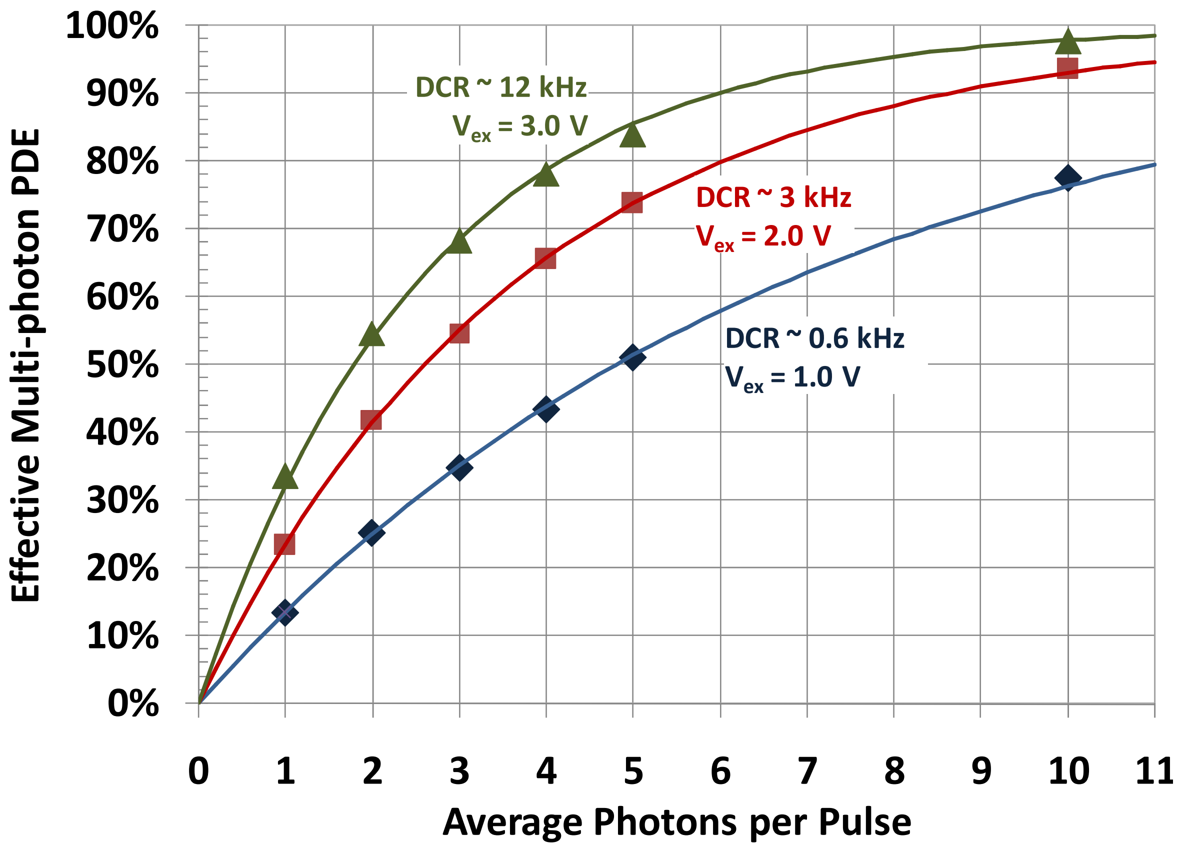
**DCR (in kHz)**



- **Largest format to date developed by MIT-LL: 64 x 256 (50 µm pitch)**

# Multi-photon pulse detection efficiency (PuDE)

- **Measure PuDE as a function of mean photon number μ**
- **Good agreement with theory:** $$\text{PuDE}(\mu) = \sum_N \frac{\mu^N e^{-\mu}}{N!}\{1 - (1 - \text{PDE})^N\}$$
  - Single photon sensitivity provides high detection probability for pulses of 5 - 10 photons



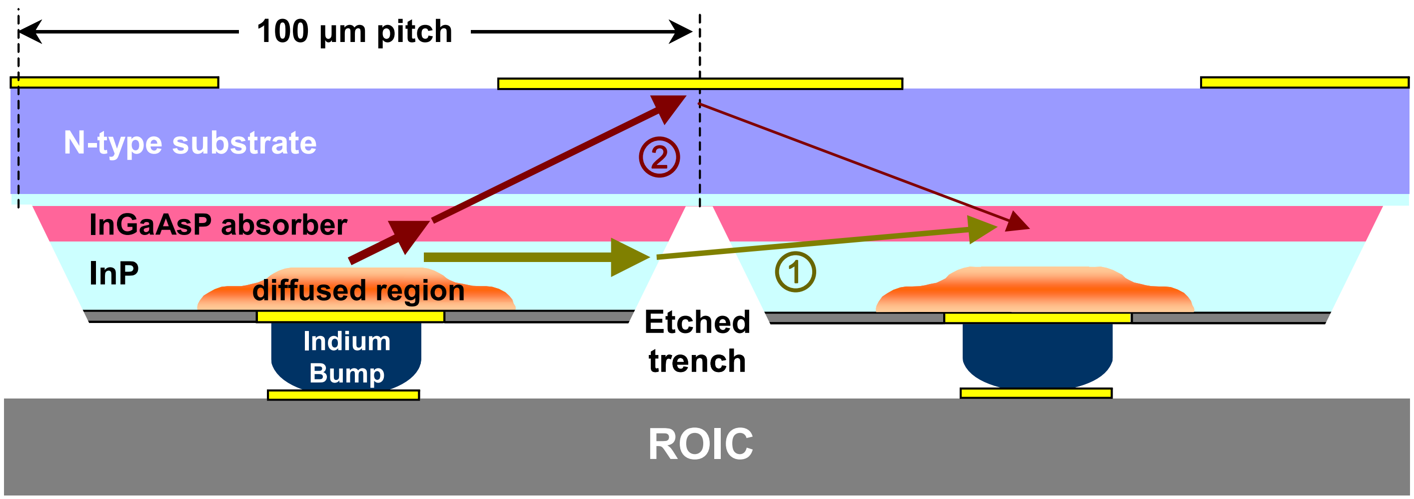

**Points:**
**experimental data**

**Solid lines:**
**theory**

# Crosstalk in SPAD arrays is challenge for scaling

- **Consider optical cross-talk contributions**
  - Avalanches can emit crosstalk photons due to hot carrier luminescence
  - Path ① : direct line-of-sight to nearest neighbor pixels
  - Path ② : reflection from back-side surface of PDA
- **Use etched trenches to mitigate line-of-sight crosstalk**

**Photo of GmAPD 32 x 32 array**



# Crosstalk as function of pixel position

- **Crosstalk falls off with distance from primary avalanche (on average)**
  - Count all events within ~ 500 µm radius and within 10 ns of primary avalanche
  - Nearest neighbor pixels show <1% crosstalk probability per pixel
  - Consistent "signature" shows that certain relative pixel positions have higher crosstalk

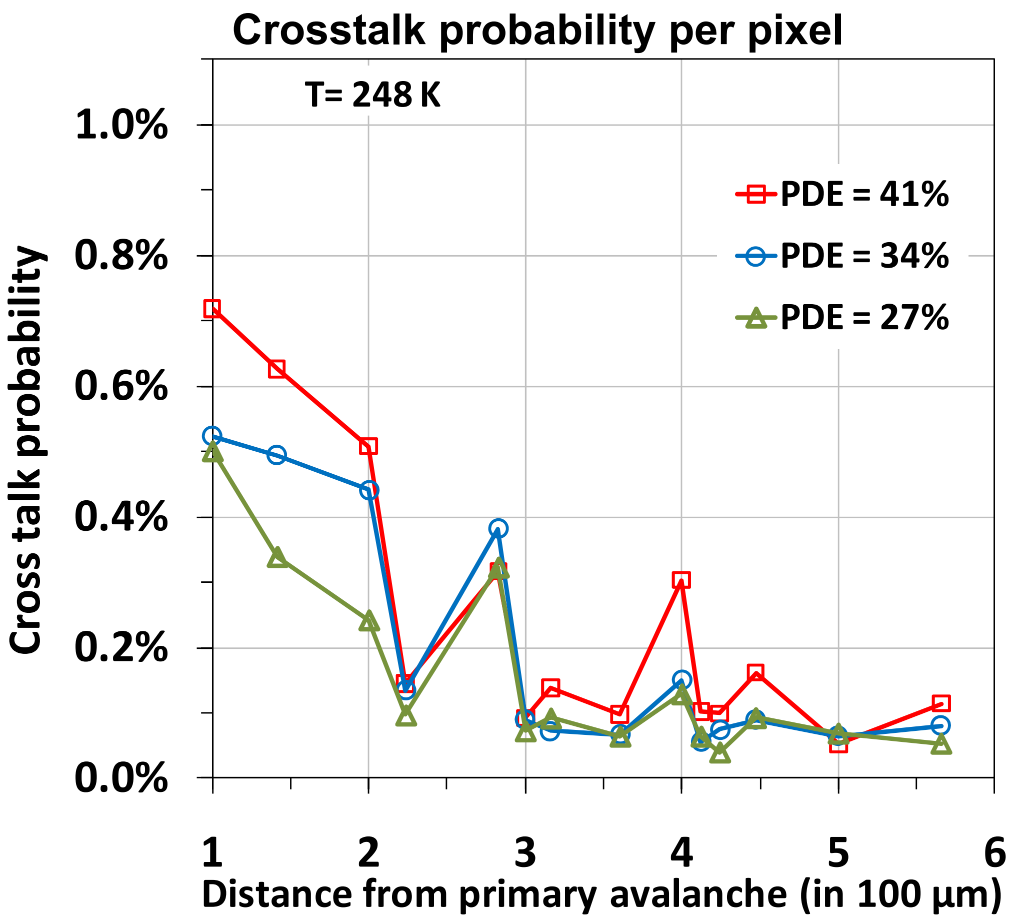



**Illustration of relative distances from primary avalanche**

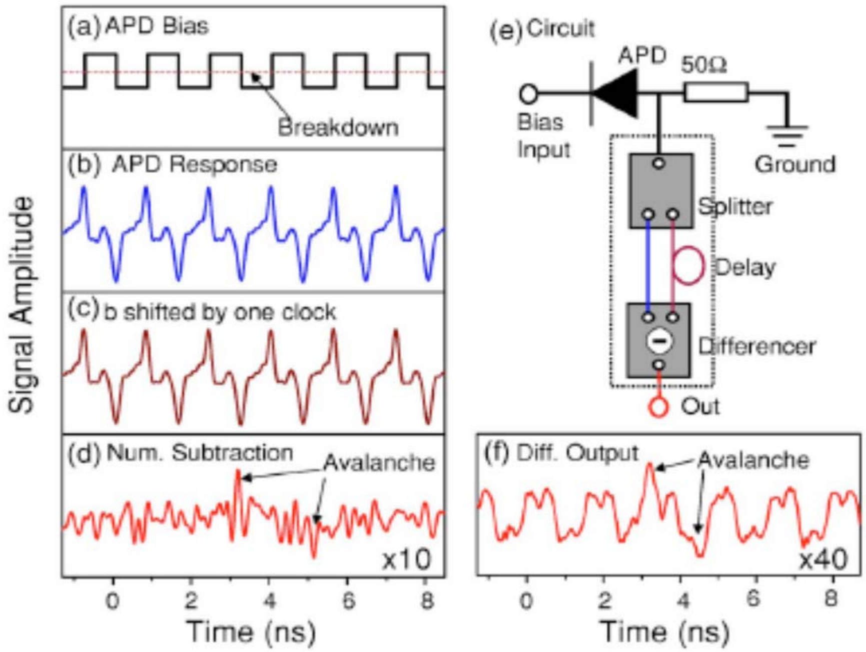
| | | | | | | |
|---|---|---|---|---|---|---|
| | | | 3 | | | |
| | 2.8 | 2.2 | 2 | 2.2 | 2.8 | |
| | 2.2 | 1.4 | 1 | 1.4 | 2.2 | |
| 3 | 2 | 1 | 0 | 1 | 2 | 3 |
| | 2.2 | 1.4 | 1 | 1.4 | 2.2 | |
| | 2.8 | 2.2 | 2 | 2.2 | 2.8 | |
| | | | 3 | | | |

# Workshop Outline

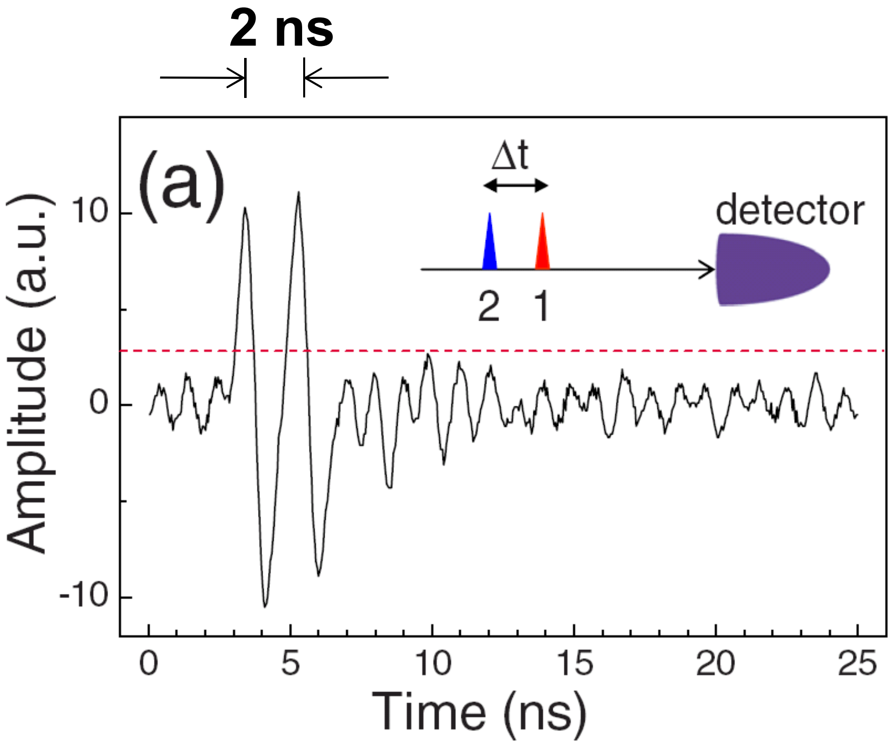
- Applications and drivers
- InGaAsP SPAD fundamentals
  - SPAD device design and performance parameters
- High-rate photon counting with InGaAsP SPADs
  - Challenges of high-rate counting: transients and afterpulsing
  - Progress in high-rate counting techniques
- Free-running operation with self-quenching NFADs
  - Integration of negative feedback
  - Self-quenching avalanche dynamics
- Scaling to large format SPAD arrays
  - Integration for focal plane arrays and FPA performance
- **Future prospects**
  - **High-rate photon counting**
  - **"Solid state photomultipliers" based on NFADs**
  - **Photon number resolution with SPADs/NFADs**
  - **Further scaling and micropixellated arrays**

# Nanosecond-scale photon counting with SPADs

- **Toshiba self-differencing technique with 1 GHz gating**
- **Key point: <u>proof-of-feasibility for SPADs counting every ~2 ns</u>**




*Z. Yuan, et al., Appl. Phys. Lett. 91, 041114 (2007)*




*A. R. Dixon, et al., Appl. Phys. Lett. 94, 231113 (2009)*

# Prospects for advances in high-rate counting

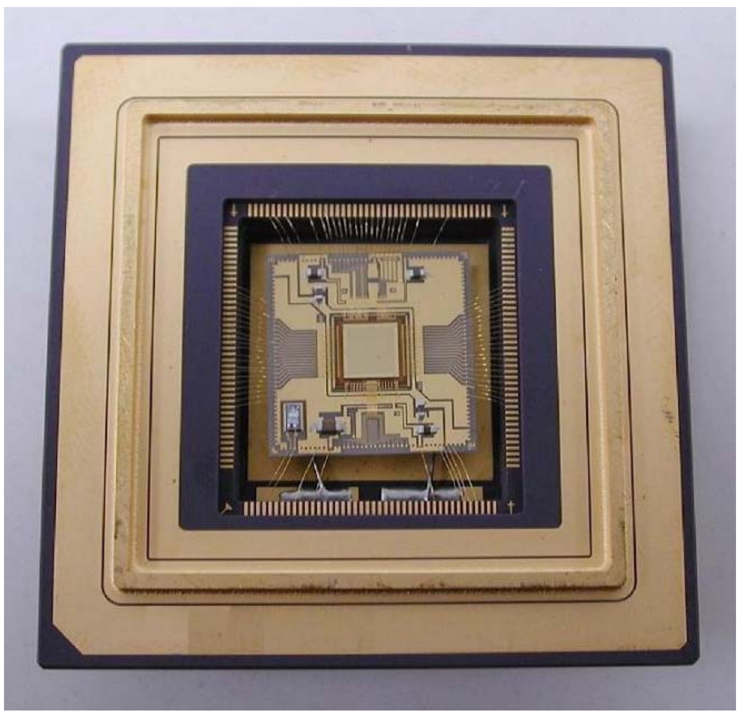
- **Timing jitter limitations**
  - for communications apps, ~100 ps jitter will limit rates to < 10 GHz
- **Inherent device bandwidth limitations**
  - same challenges as 10 GHz linear APDs (transit time / RC / avalanche build-up)
- **Challenges of non-periodic (free-running) operation**
  - All GHz-rate techniques to date require periodic operation
- **Benefits in evolving to multiplexed solutions**

# Multiplexed solutions for high-rate counting

- 1024 pixels with 250 ps timing quantization

  → **for spread optical input, ~ 4 GHz effective counting rate**
- Previous demonstrations by MIT-LL of arrays with asynchronous readout
- **…but substantial overhead in FPA complexity**

**32 x 32 FPA module**

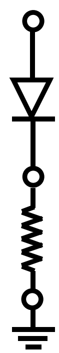


**PDE (in %)**

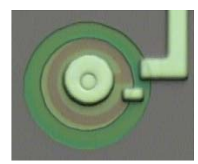
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 28 | 23 | 27 | 27 | 30 | 26 | 29 | 30 | 26 | 27 | 27 | 29 | 23 | 25 | 25 | 27 | 25 | 28 | 26 | 28 | 24 | 27 | 27 | 25 | 28 | 29 | 27 | 24 | 27 | 25 | 25 | 27 |
| **2** | 23 | 31 | 27 | 32 | 29 | 30 | 32 | 31 | 30 | 29 | 28 | 32 | 31 | 28 | 25 | 28 | 28 | 29 | 29 | 27 | 26 | 22 | 31 | 27 | 29 | 28 | 28 | 29 | 28 | 31 | 28 | 30 |
| **3** | 30 | 28 | 29 | 32 | 31 | 29 | 34 | 32 | 31 | 32 | 32 | 34 | 33 | 33 | 31 | 28 | 31 | 30 | 30 | 29 | 27 | 27 | 27 | 31 | 29 | 32 | 31 | 27 | 30 | 29 | 26 | 29 |
| **4** | 25 | 30 | 31 | 31 | 31 | 31 | 33 | 30 | 35 | 33 | 31 | 32 | 33 | 29 | 32 | 33 | 29 | 29 | 33 | 31 | 30 | 33 | 31 | 31 | 31 | 33 | 32 | 31 | 31 | 30 | 27 | 28 |
| **5** | 28 | 27 | 27 | 30 | 31 | 33 | 32 | 31 | 37 | 33 | 35 | 33 | 31 | 31 | 32 | 27 | 30 | 34 | 32 | 31 | 33 | 30 | 32 | 31 | 31 | 31 | 29 | 32 | 31 | 29 | 29 | 31 |
| **6** | 27 | 26 | 30 | 32 | 33 | 34 | 33 | 33 | 38 | 34 | 37 | 36 | 34 | 33 | 30 | 32 | 30 | 35 | 34 | 33 | 33 | 32 | 32 | 33 | 34 | 35 | 33 | 30 | 34 | 29 | 27 | 30 |
| **7** | 29 | 30 | 29 | 30 | 29 | 33 | 36 | 38 | 33 | 35 | 34 | 38 | 34 | 33 | 33 | 35 | 35 | 31 | 33 | 28 | 36 | 31 | 36 | 32 | 34 | 35 | 35 | 34 | 34 | 28 | 29 | 30 |
| **8** | 26 | 32 | 31 | 33 | 33 | 35 | 34 | 36 | 39 | 34 | 38 | 35 | 32 | 34 | 35 | 34 | 37 | 36 | 35 | 37 | 36 | 37 | 36 | 38 | 34 | 36 | 34 | 31 | 37 | 32 | 30 | 33 |
| **9** | 31 | 34 | 34 | 33 | 31 | 33 | 34 | 36 | 37 | 33 | 37 | 34 | 35 | 37 | 34 | 36 | 36 | 40 | 37 | 39 | 39 | 36 | 36 | 34 | 37 | 35 | 37 | 34 | 34 | 30 | 30 | 32 |
| **10** | 32 | 35 | 31 | 34 | 31 | 35 | 39 | 35 | 37 | 34 | 33 | 37 | 34 | 33 | 36 | 35 | 38 | 38 | 37 | 39 | 38 | 36 | 39 | 38 | 39 | 35 | 35 | 35 | 33 | 35 | 32 | 33 |
| **11** | 29 | 32 | 32 | 31 | 35 | 35 | 37 | 38 | 39 | 35 | 40 | 37 | 39 | 40 | 38 | 35 | 39 | 41 | 40 | 37 | 37 | 37 | 39 | 38 | 39 | 39 | 34 | 37 | 35 | 33 | 32 | 32 |
| **12** | 33 | 33 | 33 | 35 | 37 | 37 | 37 | 37 | 35 | 36 | 37 | 39 | 37 | 36 | 40 | 41 | 43 | 39 | 39 | 40 | 34 | 39 | 40 | 40 | 38 | 39 | 38 | 41 | 35 | 36 | 30 | 34 |
| **13** | 31 | 34 | 34 | 34 | 38 | 38 | 40 | 39 | 36 | 37 | 40 | 35 | 39 | 37 | 40 | 39 | 40 | 38 | 42 | 37 | 43 | 41 | 38 | 42 | 41 | 39 | 36 | 39 | 38 | 36 | 32 | 34 |
| **14** | 35 | 36 | 35 | 36 | 38 | 40 | 41 | 39 | 37 | 35 | 41 | 41 | 41 | 40 | 40 | 40 | 44 | 45 | 40 | 38 | 39 | 38 | 39 | 40 | 42 | 42 | 40 | 39 | 39 | 37 | 32 | 32 |
| **15** | 34 | 31 | 38 | 37 | 39 | 41 | 45 | 37 | 38 | 34 | 41 | 38 | 40 | 40 | 42 | 38 | 44 | 39 | 40 | 42 | 40 | 39 | 39 | 42 | 42 | 43 | 45 | 41 | 40 | 37 | 34 | 33 |
| **16** | 33 | 33 | 40 | 37 | 37 | 42 | 45 | 42 | 38 | 37 | 36 | 42 | 39 | 41 | 42 | 43 | 42 | 43 | 43 | 45 | 41 | 43 | 40 | 39 | 43 | 43 | 41 | 41 | 41 | 38 | 37 | 35 |
| **17** | 38 | 36 | 38 | 37 | 39 | 44 | 42 | 40 | 40 | 38 | 36 | 43 | 44 | 43 | 41 | 43 | 42 | 41 | 44 | 44 | 42 | 37 | 39 | 43 | 39 | 42 | 44 | 44 | 39 | 39 | 36 | 35 |
| **18** | 35 | 39 | 41 | 37 | 37 | 44 | 43 | 42 | 41 | 39 | 39 | 39 | 41 | 47 | 40 | 43 | 40 | 44 | 45 | 44 | 44 | 44 | 43 | 37 | 43 | 43 | 42 | 42 | 39 | 38 | 37 | 37 |
| **19** | 35 | 39 | 37 | 38 | 41 | 40 | 43 | 40 | 43 | 41 | 38 | 41 | 43 | 42 | 46 | 43 | 45 | 43 | 44 | 44 | 48 | 45 | 42 | 44 | 41 | 42 | 41 | 42 | 43 | 42 | 34 | 37 |
| **20** | 34 | 41 | 39 | 40 | 41 | 43 | 42 | 42 | 44 | 44 | 40 | 38 | 42 | 44 | 45 | 50 | 46 | 44 | 47 | 47 | 47 | 46 | 42 | 45 | 46 | 41 | 38 | 46 | 40 | 41 | 35 | 39 |
| **21** | 37 | 40 | 38 | 41 | 40 | 45 | 41 | 43 | 40 | 44 | 41 | 45 | 43 | 42 | 43 | 44 | 42 | 47 | 45 | 44 | 47 | 47 | 48 | 45 | 45 | 43 | 41 | 43 | 38 | 43 | 35 | 37 |
| **22** | 39 | 35 | 40 | 42 | 40 | 43 | 43 | 43 | 46 | 45 | 43 | 39 | 46 | 46 | 41 | 44 | 42 | 47 | 46 | 42 | 47 | 48 | 44 | 46 | 47 | 45 | 46 | 45 | 42 | 43 | 36 | 35 |
| **23** | 34 | 37 | 41 | 42 | 41 | 40 | 44 | 43 | 47 | 44 | 49 | 44 | 47 | 45 | 42 | 47 | 45 | 45 | 46 | 45 | 42 | 44 | 48 | 48 | 45 | 48 | 44 | 44 | 43 | 46 | 39 | 41 |
| **24** | 36 | 40 | 42 | 43 | 43 | 38 | 44 | 41 | 44 | 50 | 46 | 48 | 43 | 47 | 43 | 46 | 45 | 46 | 46 | 45 | 44 | 48 | 48 | 49 | 50 | 48 | 50 | 49 | 45 | 46 | 38 | 41 |
| **25** | 34 | 38 | 42 | 43 | 40 | 43 | 45 | 45 | 44 | 44 | 47 | 43 | 46 | 47 | 50 | 50 | 44 | 46 | 51 | 44 | 49 | 52 | 51 | 48 | 52 | 47 | 48 | 46 | 45 | 45 | 38 | 45 |
| **26** | 39 | 39 | 40 | 39 | 41 | 43 | 42 | 45 | 45 | 46 | 48 | 53 | 51 | 50 | 46 | 50 | 50 | 52 | 48 | 52 | 50 | 46 | 48 | 48 | 49 | 46 | 49 | 49 | 46 | 46 | 35 | 41 |
| **27** | 37 | 38 | 45 | 39 | 46 | 43 | 42 | 42 | 46 | 43 | 47 | 45 | 50 | 46 | 48 | 46 | 47 | 50 | 46 | 50 | 52 | 52 | 49 | 48 | 51 | 44 | 48 | 46 | 46 | 44 | 38 | 42 |
| **28** | 38 | 40 | 39 | 44 | 41 | 47 | 46 | 45 | 46 | 42 | 46 | 48 | 45 | 48 | 47 | 48 | 49 | 51 | 49 | 49 | 50 | 54 | 44 | 47 | 48 | 47 | 52 | 43 | 44 | 41 | 38 | 41 |
| **29** | 37 | 38 | 40 | 40 | 40 | 43 | 44 | 42 | 47 | 42 | 42 | 39 | 43 | 49 | 46 | 46 | 50 | 48 | 47 | 46 | 50 | 48 | 45 | 49 | 46 | 49 | 38 | 47 | 43 | 44 | 42 | 43 |
| **30** | 37 | 38 | 37 | 39 | 41 | 41 | 43 | 40 | 45 | 43 | 42 | 42 | 43 | 41 | 44 | 47 | 45 | 44 | 47 | 45 | 46 | 47 | 46 | 46 | 42 | 43 | 44 | 46 | 43 | 38 | 39 | 43 |
| **31** | 38 | 35 | 37 | 40 | 39 | 43 | 40 | 40 | 42 | 45 | 42 | 43 | 42 | 43 | 43 | 48 | 44 | 45 | 43 | 46 | 44 | 44 | 43 | 47 | 41 | 41 | 47 | 42 | 42 | 43 | 39 | 41 |
| **32** | 34 | 41 | 39 | 38 | 42 | 38 | 43 | 40 | 46 | 44 | 44 | 43 | 42 | 45 | 41 | 43 | 41 | 43 | 44 | 43 | 43 | 43 | 43 | 44 | 38 | 43 | 43 | 41 | 44 | 42 | 40 | 40 |

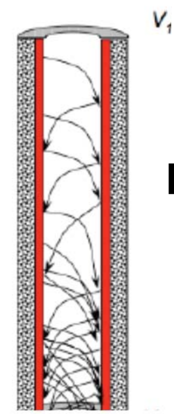
# NFADs as solid state photomultiplier (SSPM)

- **Single NFAD device independently avalanches, self-quenches, and resets**
- **NFADs exhibit reasonably uniform pulse responses**
- **Connect a “matrix” of NFAD devices in parallel**
  **→ “solid state” equivalent to microchannel plate (MCP) photomultiplier**

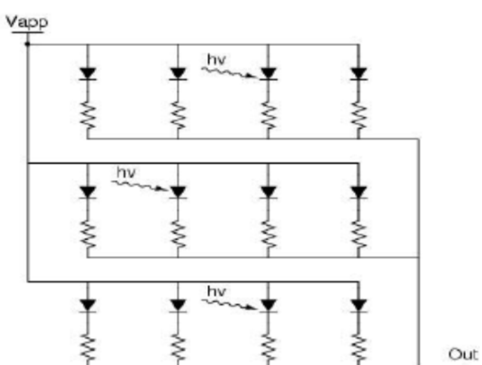


**Single NFAD**

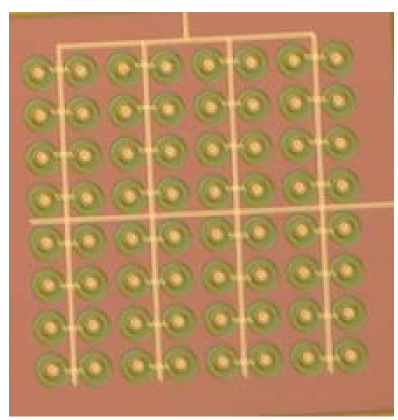




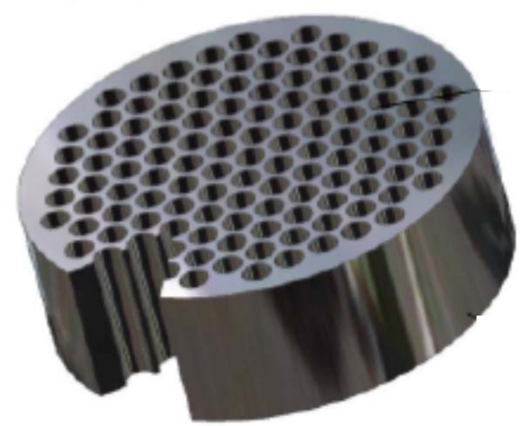

**MCP “pore”**




A.G. Stewart et al.,
SPIE 61190A (2006)



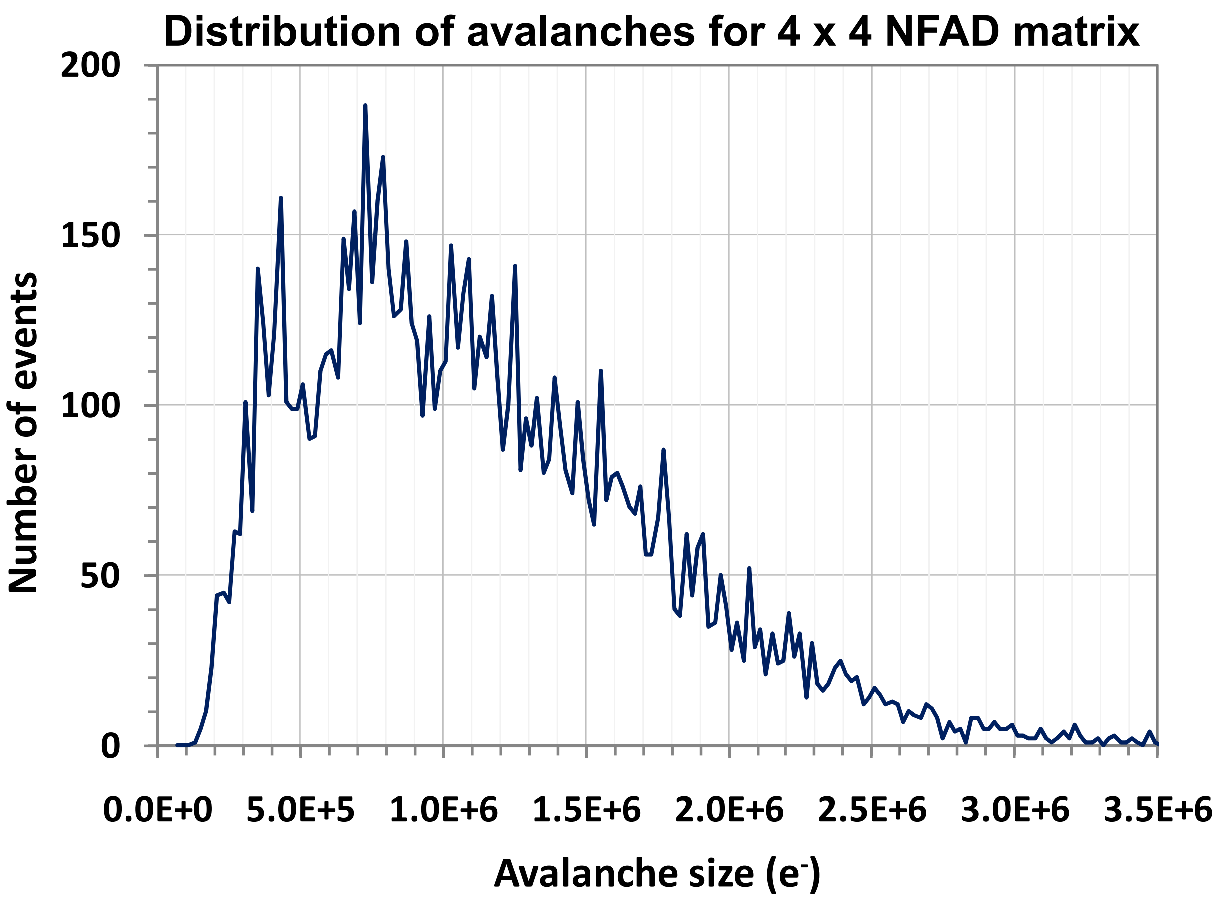
**8 x 8 matrix of InP NFADs**

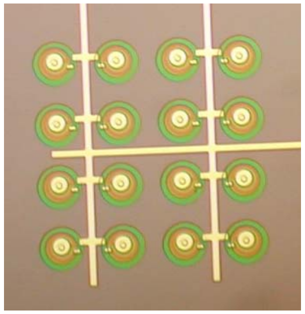


**MCP e$^-$ multiplier**

# First demonstration of NFADs as SSPM

- **“Matrix” of NFADs can provide photon number resolution**
  - Measured distribution of avalanche response peaks shows multi-avalanche structure

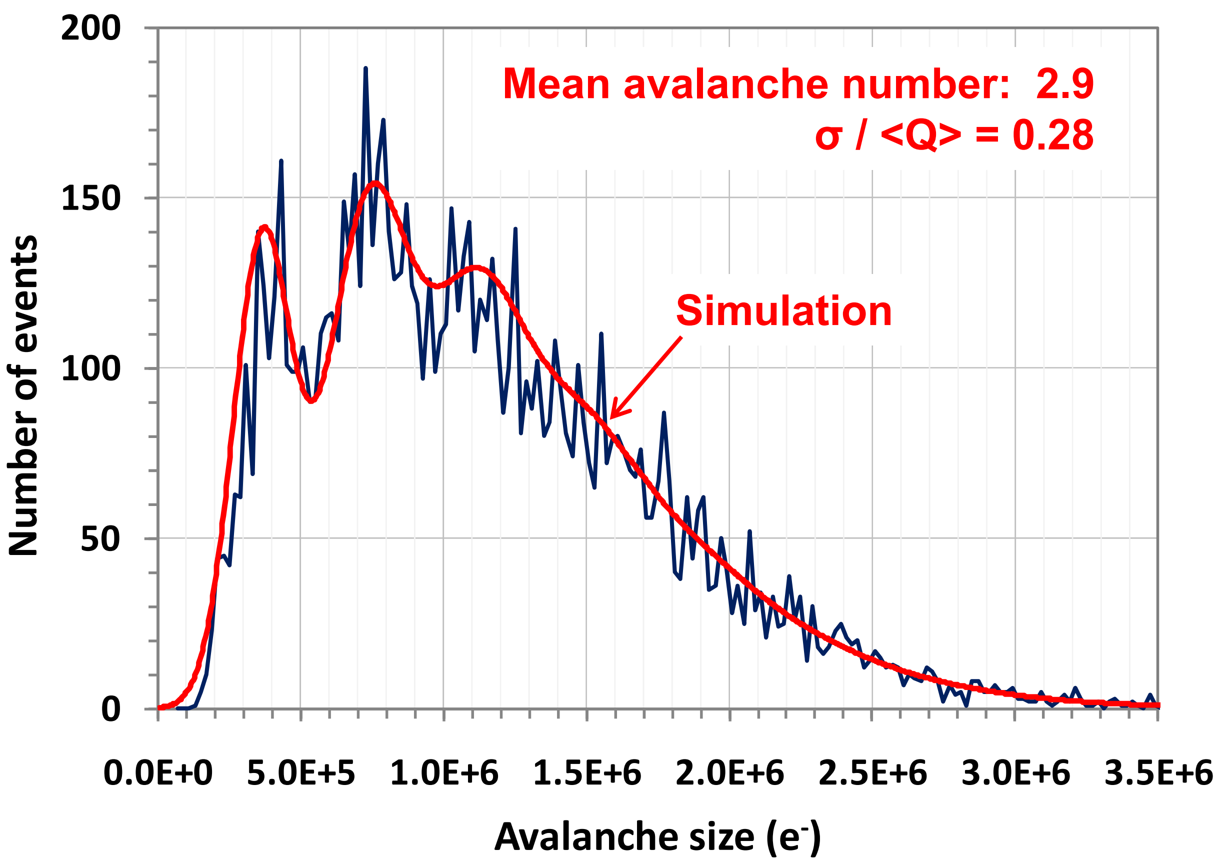


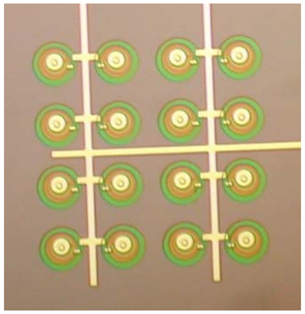



**4x4 NFAD matrix**

# First demonstration of NFADs as SSPM

- **"Matrix" of NFADs can provide photon number resolution**
  - Measured distribution of avalanche response peaks shows multi-avalanche structure
- **Simple model provides very good description of response**
  - Assume Gaussian distribution for peak height variation (σ / ⟨Q⟩ = 0.28)
  - Use Poisson statistics for incident photon number



4x4 NFAD matrix

# First demonstration of NFADs as SSPM

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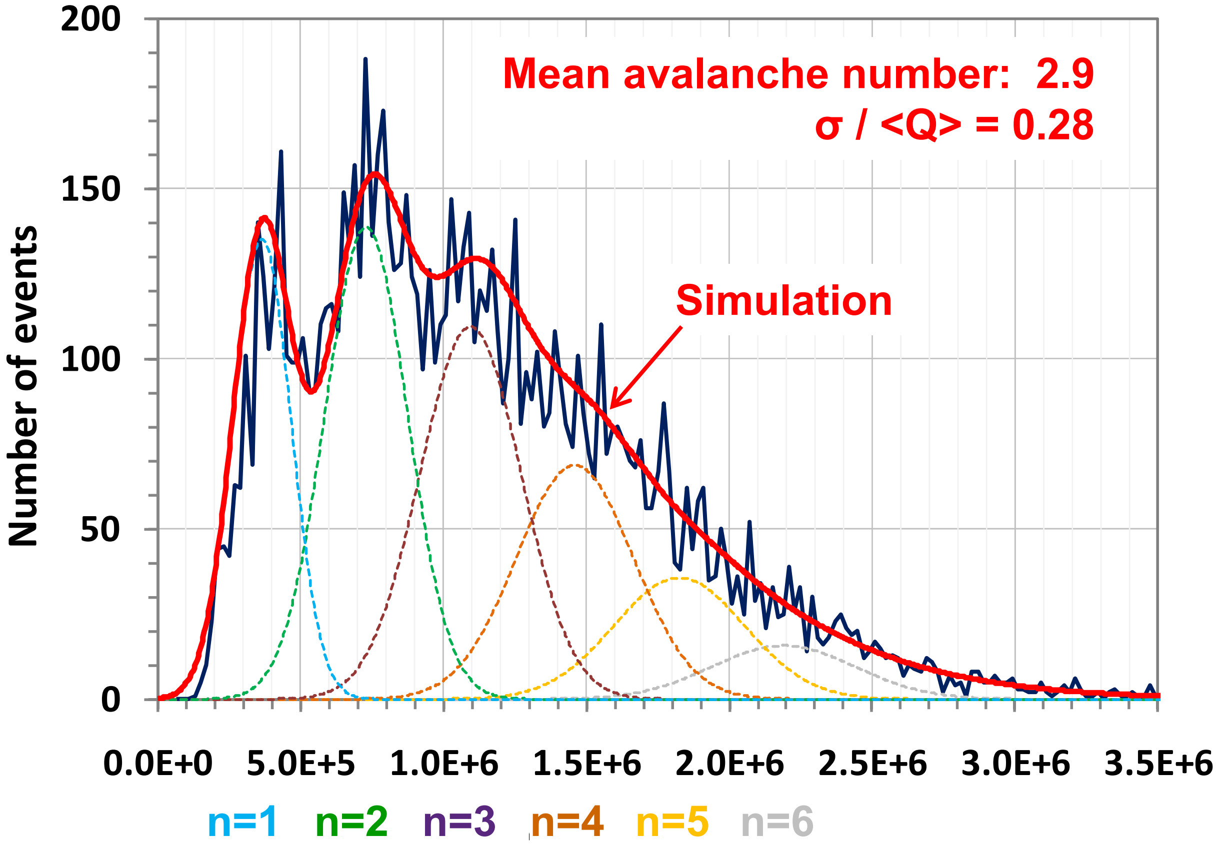
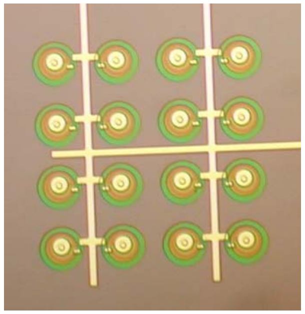
  - Assume Gaussian distribution for peak height variation (σ / ⟨Q⟩ = 0.28)
  - Use Poisson statistics for incident photon number



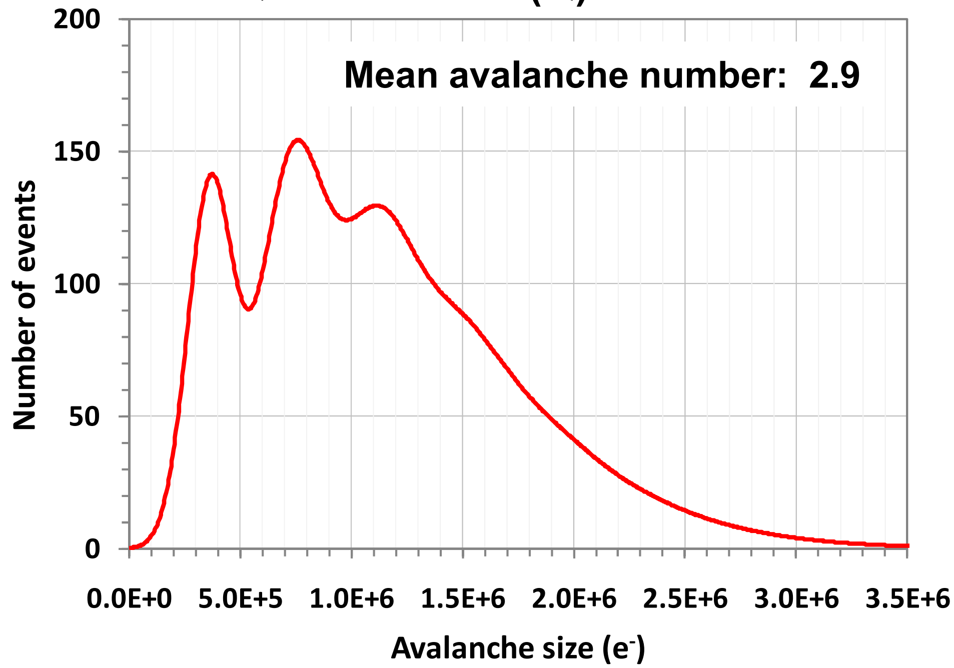
**4x4 NFAD matrix**

**"n"-avalanche peaks spaced in increments of $3.65 \times 10^5$ e$^-$**

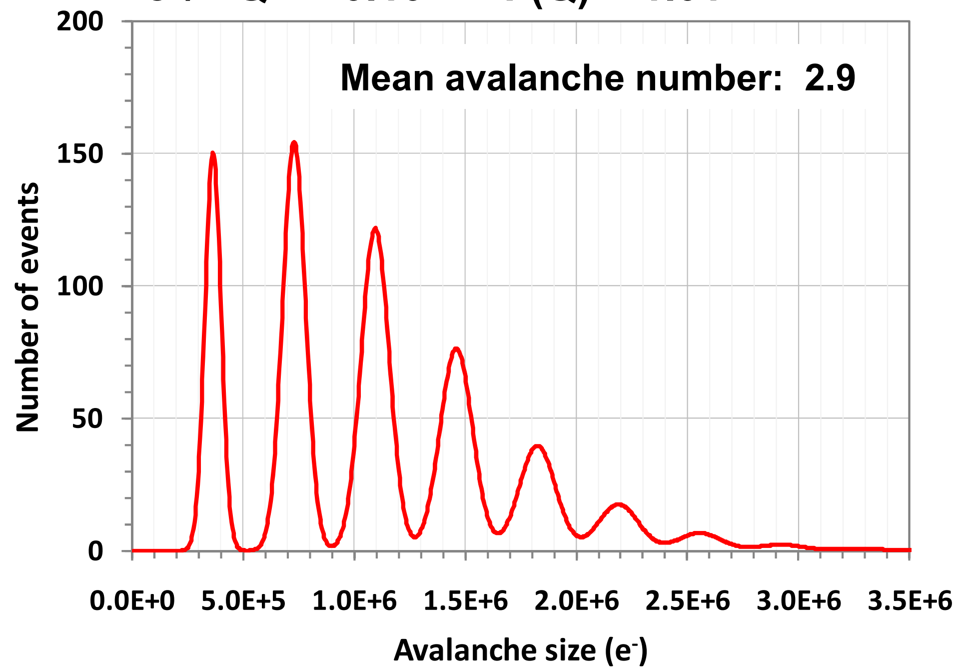
# Achieving better photon number resolution

- **Better photon number resolution will require more uniform avalanches**
  - Fully resolved peaks between n = 1 and n = 2 requires σ / <Q> ~ 0.10
- **Need further tailoring of feedback and reduction of parasitics**
  - Also work to improve device uniformity
- **Also lots of work to do on fill factor**

**σ / <Q> = 0.28 → F(Q) ~ 1.08**

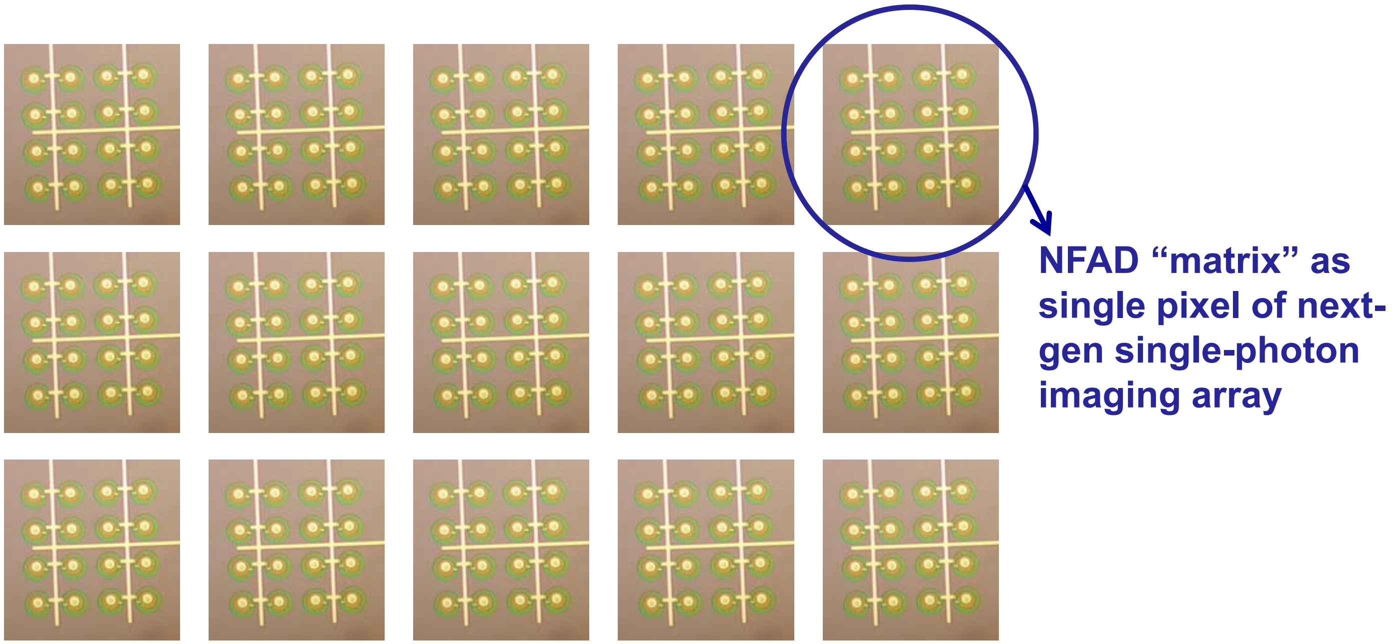



**σ / <Q> = 0.10 → F(Q) ~ 1.01**



# Potential for next-generation NFAD imager

- **Next-gen single-photon imager with NFAD "matrix" at each pixel**
  - Provide pixel-level photon number resolution (PNR)
  - Degree of PNR determined by number of matrixed elements

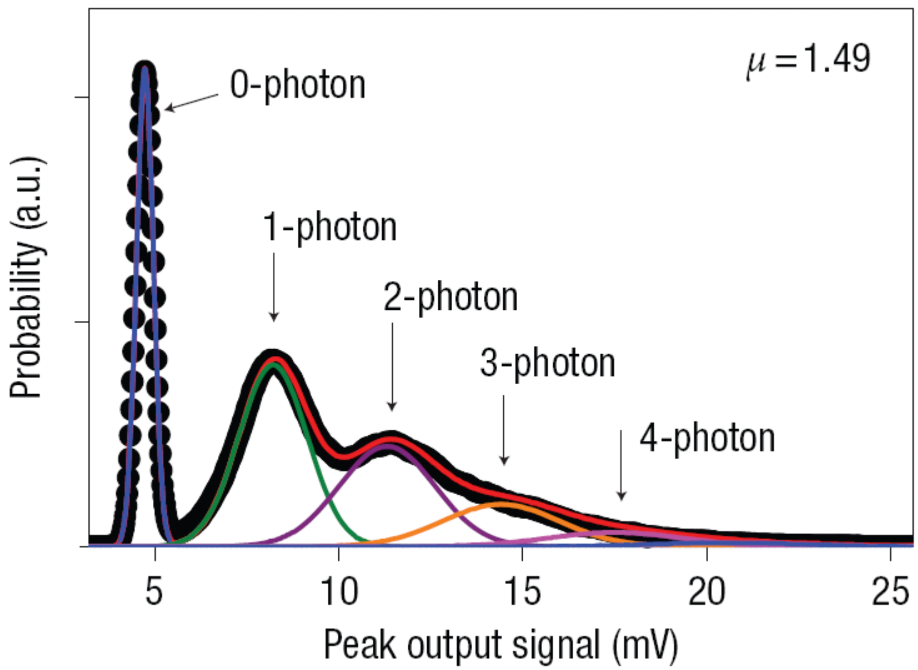



- **Also pursuing "active" NFADs with "two-state" feedback element**
  - High resistance for quenching, low resistance for re-charging

# Photon number resolution with self-differencing

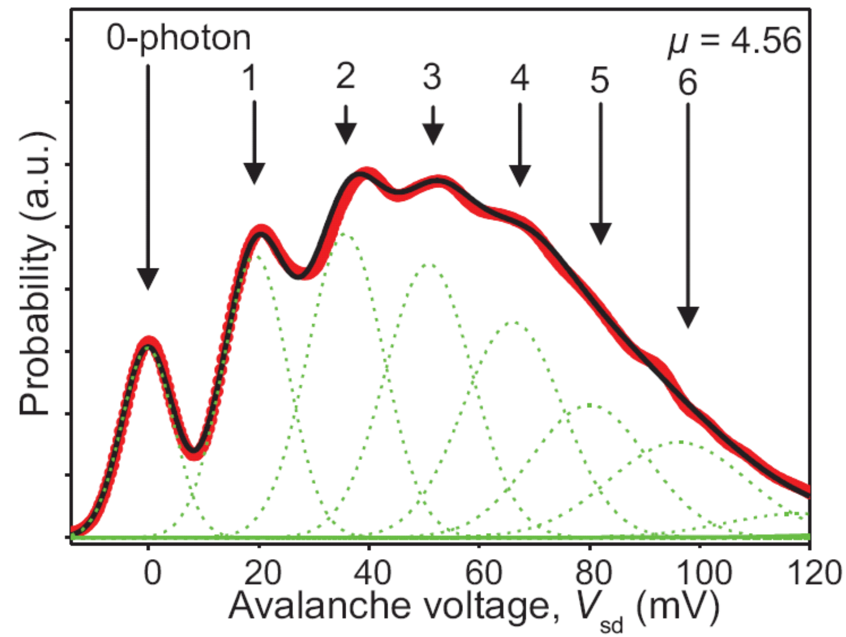
- **SPAD can have analog response with sensitivity to photon number**
  - Demonstrated with Toshiba self-differencing circuit
  - Histogram shape dictated by (i) Poisson distributed input, (ii) σ of charge flow per photon
- **Key is to restrict avalanche flow** (very short sub-ns gates in this case)

**InGaAs/InP SPAD**




*Kardynal, et al., Nature Photonics, 15 June 2008*
*doi:10.1038/nphoton.2008.101*

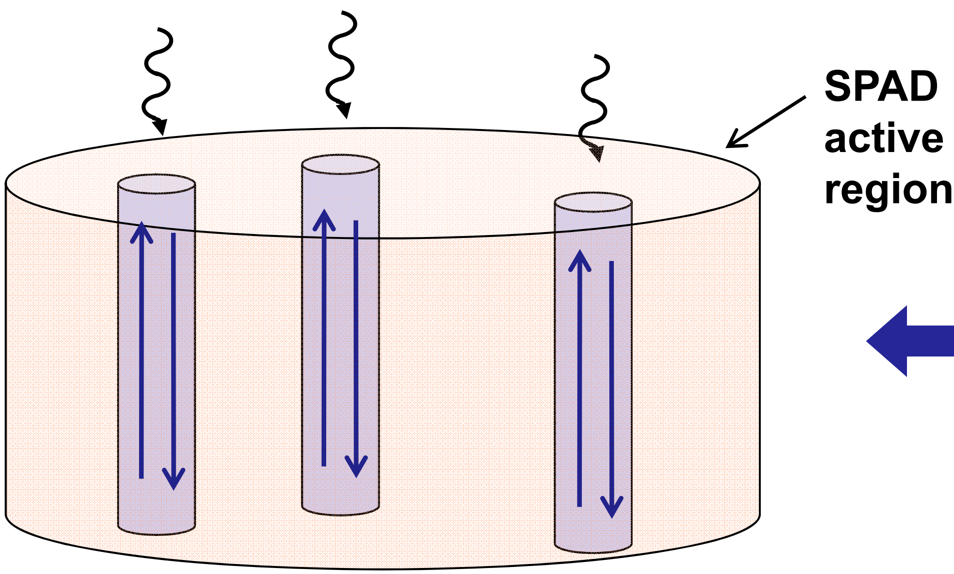
**Silicon SPAD**




*Single Photon Workshop 2011, 29 June 2011*
*Braunschweig, Germany*

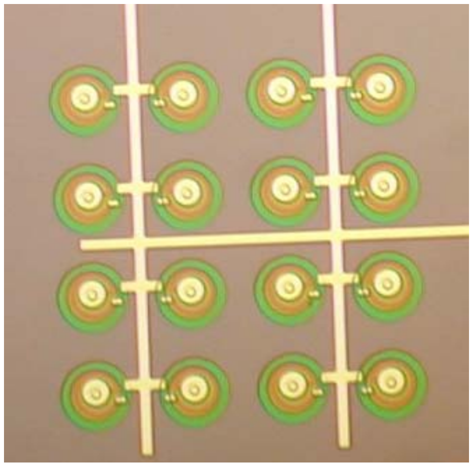
# PNR through fabricated NFAD micro-pixellation

- **PNR in discrete SPAD likely due to individual avalanche "filaments"**
  - Sufficient filament uniformity with fast quenching before lateral spreading
- **NFAD matrix provides similar "micropixellation" by fabrication**
  - Sufficient avalanche uniformity from negative feedback
- **Avalanche filament control converges on linear mode operation**




**Distinct avalanche "filaments" providing PNR is single SPAD**

**NFAD matrix**

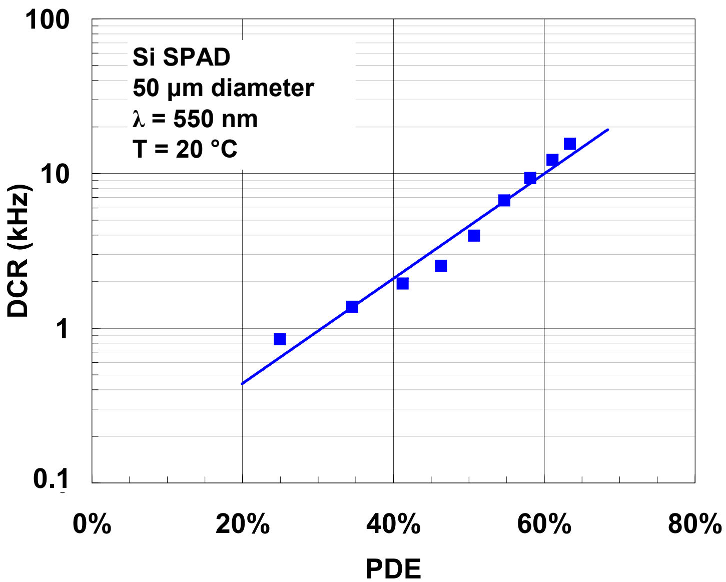


**NFAD matrix can provide "filaments" by design**

# Comparison of InGaAsP SPADs and Si SPADs

- **What can we project for InGaAsP SPADs based on more mature Si SPADs?**
- **Compare at different temperatures to compensate for difference in $E_g$**
  - Si outperforms InP by ~10X in DCR at same PDE
  - Best hold-off times for Si ~ 10 ns (1% afterpulsing, 20°C), ~10X better than InP at -60°C
    - Afterpulsing comparison is approximate due to strong circuit-dependence

**Thin absorber Si SPAD data for optimal wavelength**




*Data from M. Ghioni and S. Cova, Politecnico di Milano*

| | Si | InGaAs/InP |
|---|---|---|
| Temperature | 20 °C | – 70 °C |
| Diameter | 50 µm | |
| Wavelength | 550 nm | 1550 nm |
| DCR vs PDE | 10 kHz at 60%<br>2 kHz at 40%<br>0.5 kHz at 20% | 10 kHz at 40%<br>2 kHz at 20%<br>1 kHz at 10% |
| Min hold-off for 1% AP (free-run) | ~ 10 ns | ~ 100 ns |
| Jitter (FWHM) | 30 – 50 ps | 50 – 100 ps |

# Summary: What lies ahead for InGaAsP SPADs?

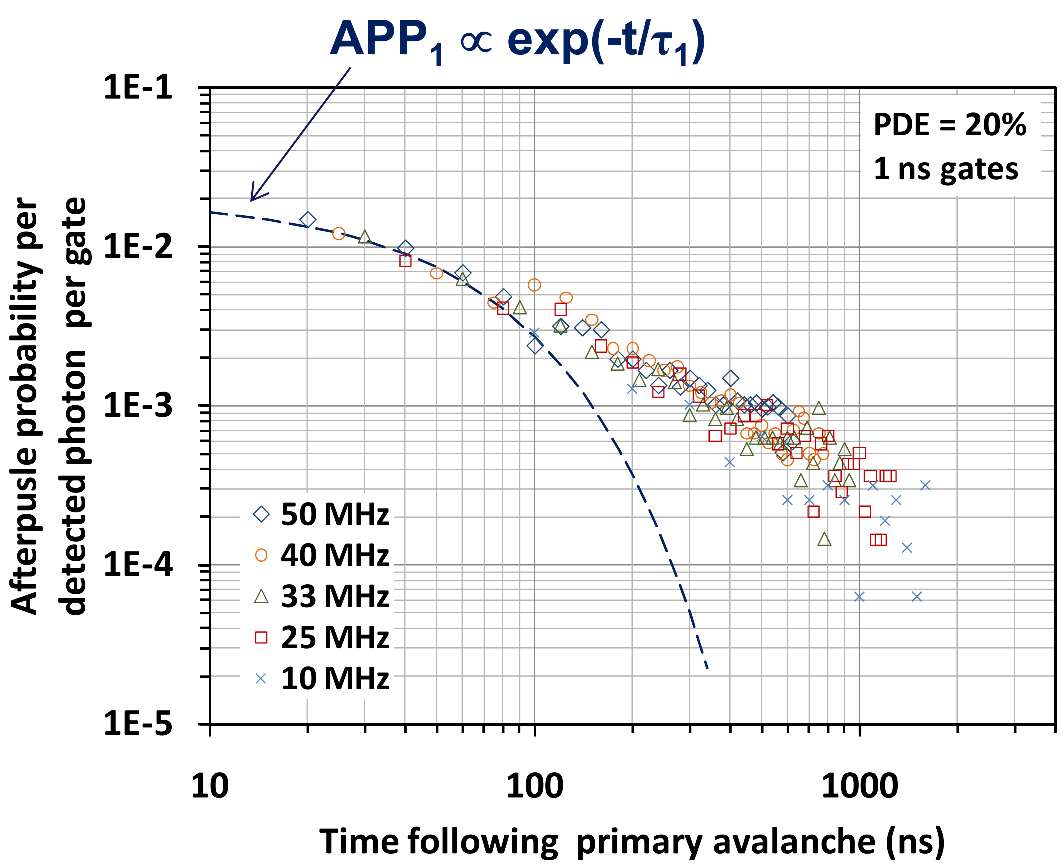
Princeton Lightwave

- **High-rate counting up to ~5 GHz for discrete detectors**
  - 0.5 GHz counting demonstrated with sub-ns periodic gating
  - Discrete detector counting limited to ~few GHz by fundamental APD dynamics
  - To reach even higher rates, use multiplexed solutions
- **Potential for analog behavior with SPADs/NFADs**
  - Photon number resolution (PNR) is feasible even in discrete SPADs
  - Micropixellation provides potential for more extensive PNR
  - Convergence of linear mode and Geiger-mode through negative feedback
- **Scaling to larger format arrays (e.g., Mpixel) is achievable**
  - Increased pixel count is challenging, but no fundamental limits
  - Further pitch reduction increasingly difficult due to single-photon crosstalk
- **Improvement in basic parameters requires materials advances**
  - DCR and afterpulsing directly related to material defect density
  - Higher PDE accessible if DCR and afterpulsing are tolerable at higher bias
- **Smart design concepts will progress faster than materials improvements**

# BACK-UP SLIDES

# Legacy approach to afterpulse fitting

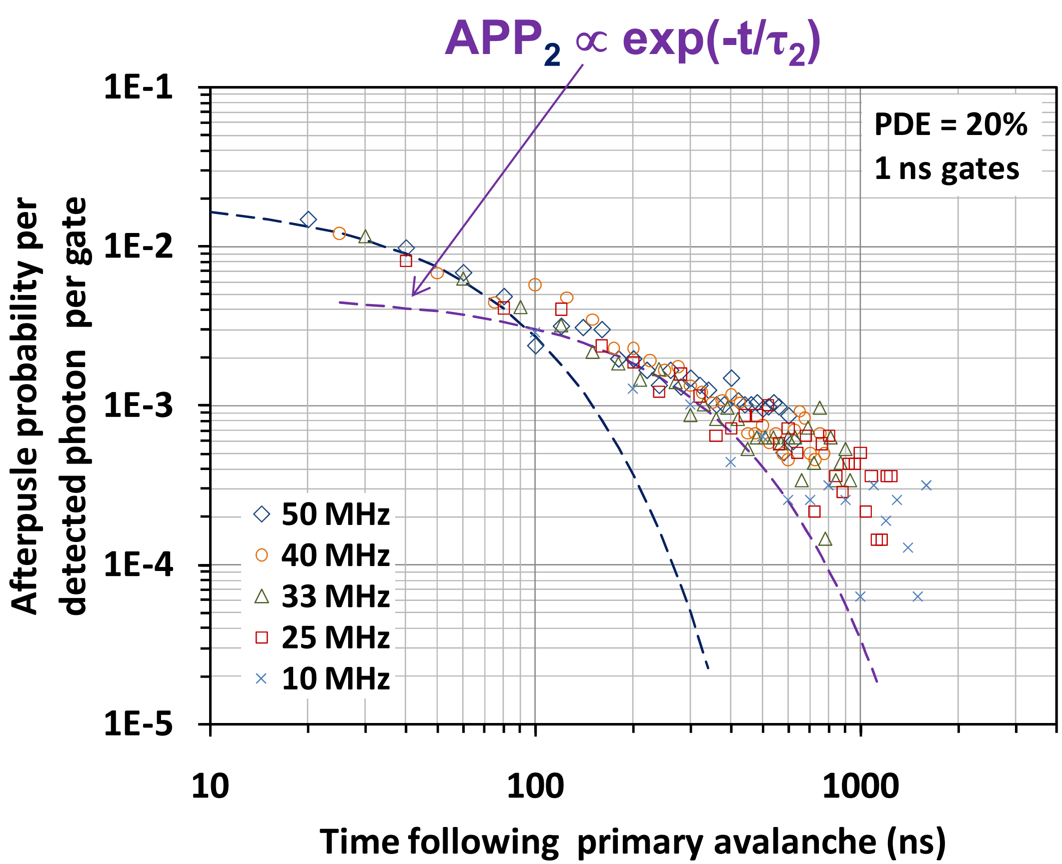
- **Try to fit afterpulse probability (APP) data with exponential fit**
  - Physically motivated by assumption of single dominant trap




**Single exponential curve generally fits range of ~5X in time**

# Legacy approach to afterpulse fitting

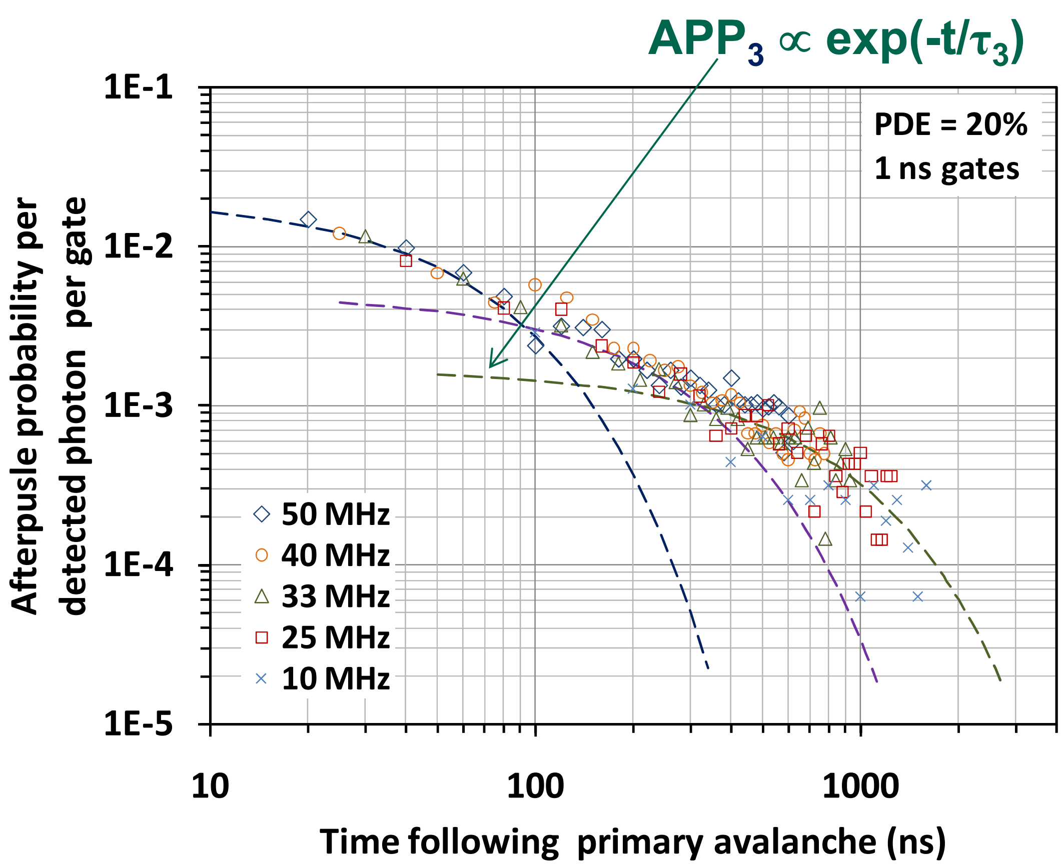
- **Try to fit afterpulse probability (APP) data with exponentials**
  - Physically motivated by assumption of single dominant trap
- **Single exponential not sufficient; assume second trap**




**Single exponential curve generally fits range of ~5X in time**

# Legacy approach to afterpulse fitting

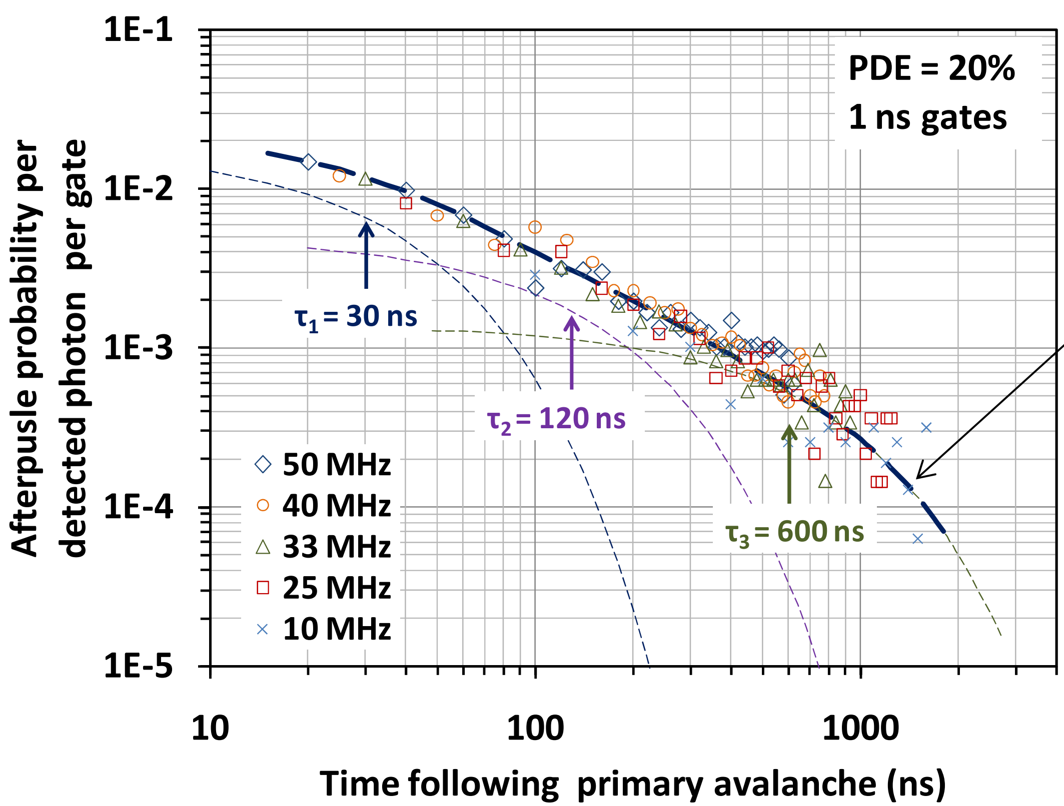
- **Try to fit afterpulse probability (APP) data with exponentials**
  - Physically motivated by assumption of single dominant trap
- **Single exponential not sufficient; assume second trap**
- **Still need third exponential to fit full data set**




**Single exponential curve generally fits range of ~5X in time**

# Legacy approach to afterpulse fitting

- **Can always achieve reasonable fit with several exponentials**
- **…but <u>choice of time constants is completely arbitrary!</u>**
  → depends on range of times used in data set
- **Our assertion: <u>No physical significance to time constants in fitting</u>**
  → simply minimum set of values to fit the data set in question



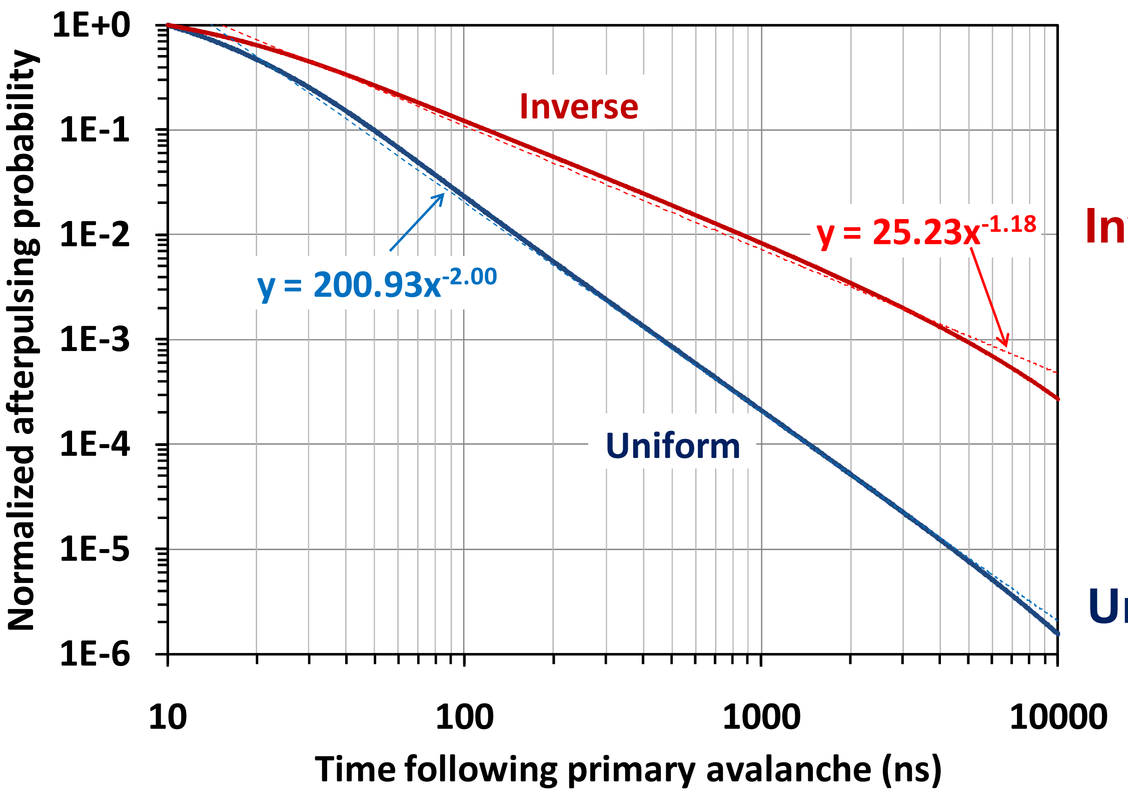

$$APP = C_1 \exp(-t/\tau_1) + C_2 \exp(-t/\tau_2) + C_3 \exp(-t/\tau_3)$$

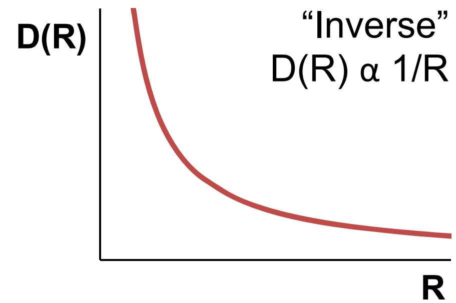
# Modeling results for APP

- **Develop model for APP with distribution of detrap rates R ≡ 1/τ**
  - APP related to change in trap occupation: dN/dt ~ R exp(-t R)
  - Integrate over detrapping rate distribution D(R)

  **→ APP ~ ∫ dR D(R) R exp(-t R)**

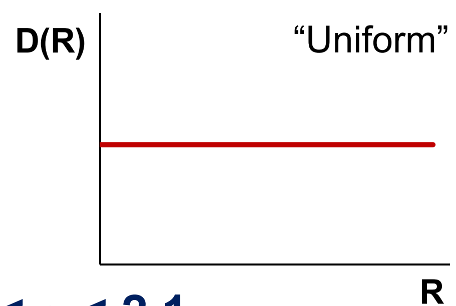
- **APP behavior fit well by $T^{-\alpha}$ for 10 ns to 10 μs**




**Inverse D(R):**

**$T^{-\alpha}$ with 1.05 < α < 1.30**




**Uniform D(R):**

**$T^{-\alpha}$ with 1.9 < α < 2.1**